MATH 4331/6312
Introduction to Real Analysis
Fall 2019

First name: _____ **Last name:** _____ | **Points:**

Assignment 3, due Thursday, September 12, 10am

Please staple this cover page to your homework. Circle your course number, Math 4331 or 6312. When asked to prove something, make a careful step–by–step argument. You can quote anything we covered in class or in preceding homework in support of your reasoning.

Problem 1

Show that the Cantor set has empty interior. Hint: The construction of the Cantor set can be described in the following way: $C_0 = [0,1]$, $C_1 = [0,1] \setminus (1/3, 2/3)$, $C_2 = C_1 \setminus ((1/9, 2/9) \cup (4/9, 5/9) \cup (7/9, 8/9))$ etc., so at each step open intervals are removed. Note that $(4/9, 5/9)$ appears only for notational convenience.

Problem 2

Let A and B be subsets of $\mathbb{R}^n$, $A \subset B$. The set A is dense in B if B is contained in $\overline{A}$.

   a. Show that for $n = 1$, the set of irrational numbers, $\mathbb{R} \setminus \mathbb{Q}$, is dense in $\mathbb{R}$.

   b. Hence, show that $\mathbb{Q}$ has empty interior.

Problem 3

Let f be defined on $\mathbb{R}$ by $f(x) = x$ if $x \in \mathbb{Q}$ and $f(x) = 0$ otherwise. Show that f is continuous at 0 but not continuous at any $a \neq 0$.

Problem 4

Let $m : \mathbb{R}^2 \to \mathbb{R}$ be defined by $m(x,y) = \max\{x, y\}$.

   a. Show that m is continuous.

   b. Hence, show that for two real-valued continuous functions f and g defined on a set $S \subset \mathbb{R}^n$, the function $h : S \mapsto \mathbb{R}$ defined by $h(x) = \max\{f(x), g(x)\}$ is continuous.