Department of Mathematics University of Houston

# MATH 4331/6312

## Introduction to Real Analysis
Fall 2019

**First name:** ____________________ **Last name:** ______________________ **Points:**

# Assignment 4, due Thursday, September 19, 10am

**Please staple this cover page to your homework. Circle your course number, Math 4331 or 6312.** When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

## Problem 1

Let $f$ be a continuous function defined on an **open** subset $S$ of $\mathbb{R}^n$. Prove that the set $\{(x_1, x_2, \ldots, x_n, y) : x \in S, y > f(x)\}$ is an open subset of $\mathbb{R}^{n+1}$. If useful, abbreviate $(x_1, x_2, \ldots, x_n, y) = (x, y)$.

## Problem 2

Show that the function $f(x) = x^p$ on defined on $\mathbb{R}$ is not uniformly continuous if $p \in \{2, 3, 4, \ldots\}$.

## Problem 3

A function $f : \mathbb{R} \to \mathbb{R}$ is called 1-periodic if $f(x) = f(x+1)$ for each $x \in \mathbb{R}$. Show that if $f$ is continuous, then it is uniformly continuous.

## Problem 4

Let $A$ be a compact subset of $\mathbb{R}^n$. Show that for any $x \in \mathbb{R}^n$, there is $a \in A$ which is closest to $x$ among the points in $A$, so for any $y \in A$, $\|y - x\| \geq \|a - x\|$. Hint: Fix $x$ and introduce a useful function on $A$ which you show to be continuous, then quote a result from class.

## Problem 5

Assume a real-valued function $f$ is continuous on $\mathbb{R}^n$ and satisfies $f(x) \geq 0$ for all $x \in \mathbb{R}^n$ as well as $\lim_{\|x\| \to \infty} f(x) = 0$, i.e. for each $\epsilon > 0$ there is $R > 0$ such that $f(x) < \epsilon$ for all $x$ with $\|x\| > R$. Show that $f$ attains its maximum value.