MATH 4331/6312

Introduction to Real Analysis
Fall 2019

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 5, due Thursday, September 26, 10am

Please staple this cover page to your homework. Circle your course number, Math 4331 or 6312. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Let $S \subset \mathbb{R}^n$ and $T \subset \mathbb{R}^m$, $f : S \to T$ and $g : T \to \mathbb{R}^k$ be uniformly continuous functions, then show that $h = g \circ f$ is uniformly continuous from S to $\mathbb{R}^k$.

Problem 2

Show that if S is a connected subset of $\mathbb{R}^n$, then the closure $\overline{S}$ is connected.

Problem 3

Let the surface of the planet Mars be represented by the sphere $S = \{(x, y, z) : x^2 + y^2 + z^2 = 1\}$ and assume the temperature $T : S \to \mathbb{R}$ is a continuous function on S. Show that there is a point $(x, y, z) \in S$ such that $T(x, y, z) = T(-x, -y, -z)$.
Hint: Consider $f(x, y, z) = T(x, y, z) - T(-x, -y, -z)$, compare $f(x, y, z)$ and $f(-x, -y, -z)$.

Problem 4

Let f be a continuous function from the ball $B_1 = \{(x, y) : x^2 + y^2 \leq 1\}$ in $\mathbb{R}^2$ to $\mathbb{R}$. Show that this $f : B_1 \to \mathbb{R}$ cannot be one-to-one.