Department of Mathematics University of Houston

MATH 4331/6312

**Introduction to Real Analysis**
**Fall 2019**

**First name:** ____________________ **Last name:** ______________________ **Points:**

# Assignment 7, due Thursday, October 24, 8:30am

**Please staple this cover page to your homework. Circle your course number, 4331 or 6312.** When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

## Problem 1

If a function $f : [a, b] \to \mathbb{R}$ is Lipschitz-continuous with Lipschitz constant $C$, then prove that for any partition $P$ of $[a, b]$, we have $U(f, P) - L(f, P) \leq C(b - a)\text{mesh}(P)$.

## Problem 2

Show that if a real-valued function $f$ is bounded and Riemann integrable on $[a, b]$, so is $|f|, |f|(x) = |f(x)|$.

## Problem 3

Show that if $f$ and $g$ are real-valued, bounded and Riemann integrable on $[a, b]$ and $f(x) \leq g(x)$ for each $x \in [a, b]$, then

$$\int_a^b f(x)dx \leq \int_a^b g(x)dx\,.$$

## Problem 4

Prove the mean value theorem for integrals: If a real-valued function $f$ is continuous on $[a, b]$, then there is $c \in (a, b)$ such that

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx\,.$$

## Problem 5

Let $C$ be the Cantor set as defined in class (or Assignment 3), and

$$f(x) = \begin{cases} 1, & x \in C \\ 0, & \text{else} \end{cases}$$

for $x \in [0, 1]$. Show that $f$ is Riemann integrable on $[0, 1]$ and that $\int_0^1 f(x)dx = 0$.