Department of Mathematics University of Houston

MATH 4331/6312

Introduction to Real Analysis
Fall 2019

First name: ______________________ **Last name:** ________________________ **Points:**

Assignment 8, due Thursday, October 31, 8:30am

Please staple this cover page to your homework. Circle your course number, 4331 or 6312. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

A step function $f : [a, b] \to \mathbb{R}$ is a function so that there is a partition $P = \{a, x_1, \ldots, x_{n-1}, b\}$ for which f is constant on each interval (x_{j-1}, x_j). Show that each such function is Riemann integrable.

Problem 2

Let $f : [a, b] \to \mathbb{R}$ be bounded and Riemann integrable. define $g : [a+c, b+c] \to \mathbb{R}$ by $g(x) = f(x-c)$. Show that g is Riemann integrable and that

$$\int_{a+c}^{b+c} g(x)dx = \int_a^b f(x)dx .$$

Problem 3

Let $f : [a, b] \to \mathbb{R}$ be bounded and Riemann integrable. Define for $c > 0$ a function $g : [ca, cb] \to \mathbb{R}$ by $g(x) = f(x/c)$. Show g is Riemann integrable on $[ca, cb]$ and prove the simple substitution rule

$$\int_{ca}^{cb} g(x)dx = c\int_a^b f(x)dx .$$

Problem 4

Define $\ln(y) = \int_1^y \frac{1}{x}dx$ for any $y > 0$. By using properties of integrals, show that for $a, b > 0$, $\ln(ab) = \ln a + \ln b$.

Problem 5

Let $f : \mathbb{R} \to \mathbb{R}$ be continous, and fix $c > 0$. Show that the function

$$G(x) = \frac{1}{c}\int_x^{x+c} f(t)dt$$

has a continuous derivative and compute $G'(x)$.