MATH 4332

Introduction to Real Analysis

Spring 2016

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 2, due Thursday, February 4, 2:30pm

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Show that every open set A in a metric space (X, d) is the union of closed sets.

Problem 2

Let (X, d) be a metric space and $A \subset X$. Let E be the set of all $p \in X$ for which there is a sequence $\{p_n\}_{n\in\mathbb{N}}$ with $p_n \in A$ for each $n \in \mathbb{N}$ and $\lim_{n\to\infty} p_n = p$. Show that E is the closure of A.

Problem 3

Let $\mathbb{R}$ be equipped with the usual metric and $A = \{\frac{1}{n} : n \in \mathbb{N}\}$.

a. Show that A is not compact.

b. Show that $A \cup \{0\}$ is compact.

Problem 4

Let (X, d) be a metric space and K_1, K_2, $\ldots$, K_n be compact subsets of X. Prove that $K = K_1 \cup K_2 \cup \ldots K_n$ is compact.