MATH 4331

Introduction to Real Analysis

Spring 2016

First name: __________________ **Last name:** ____________________ **Points:**

Assignment 3, due Thursday, February 11, 2:30pm

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Let (X, d) be a metric space and $K \subset X$ be compact. Prove that K is bounded.

Problem 2

Let (X, d) be a metric space, $Y \subset X$ and consider the metric space (Y, d).

a. Show that every open set U in Y has the form $U = V \cap Y$ for an open set V in X. Hint: Show this first for open balls.

b. Show that Y is compact in (Y, d) if and only if every collection of open sets $\{V_j\}_{j\in J}$ in X that covers Y has a finite subcover.

Problem 3

Find an example for two metric spaces (X, d) and (Y, ρ), a continuous function $f : X \to Y$ and a Cauchy sequence $\{p_n\}_{n\in\mathbb{N}}$ in X which is not mapped to a Cauchy sequence in Y.

Problem 4

Let (X, d) and (Y, ρ) be metric spaces. Prove that if $f : X \to Y$ is continuous, then for any set A in X with closure $\overline{A}$,

a. we have $f(\overline{A}) \subset \overline{f(A)}$

b. and this inclusion can be proper, i.e. give an example for which $f(\overline{A}) \neq \overline{f(A)}$.