Department of Mathematics University of Houston

MATH 4331

Introduction to Real Analysis

Spring 2016

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 4, due Thursday, February 18, 2:30pm

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Let $X = [0, \infty)$, equipped with the usual metric from $\mathbb{R}$. Show that the function $f(x) = \sqrt{x}$ is uniformly continuous on X. Hint: Use the inequality $\sqrt{a+b} \leq \sqrt{a} + \sqrt{b}$ for any $a, b \geq 0$ (proved by squaring both sides).

Problem 2

Let (X, d) and (Y, ρ) be metric spaces, X compact. Show that if $f : X \to Y$ is continuous, one-to-one and onto, then f^{-1} is continuous.

Problem 3

Show that the completion of a metric space (X, ρ) is compact if and only if X is totally bounded.

Problem 4

Show that $[0, 1]$, equipped with the usual metric, is not the disjoint union of a countably infinite family of nonempty closed sets $\{A_n\}_{n=1}^{\infty}$. Hint: If $U_n = A_n^{\circ}$, then explain why the complement of $\cup_{n=1}^{\infty} U_n$, $X = [0, 1] \setminus \cup_{n=1}^{\infty} U_n$ is complete (with respect to the usual metric). Next, find an integer n_0 and an open interval V such that $X \cap V \neq \emptyset$ and $X \cap V \subset A_{n_0}$. Show $U_n \cap V = \emptyset$ if $n \neq n_0$.