Department of Mathematics University of Houston

MATH 4332

Introduction to Real Analysis

Spring 2016

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 8, due Thursday, April 21, 2:30pm

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Compute the Fourier series of the following functions on $[-\pi, \pi]$:

a. $f(t) = \cos^3(t)$,

b. $f(t) = |\sin t|$,

c. $f(t) = t$.

Problem 2

If f is a 2π-periodic function with known Fourier series, let $\alpha \in \mathbb{R}$ and define g by $g(t) = f(t - \alpha)$, $t \in \mathbb{R}$. Express the Fourier series of g in terms of that of f. Use this together with Problem 1.b to find the Fourier series of $g(t) = |\cos(t)|$.

Problem 3

Show that if f is continuous and 2π-periodic with Fourier coefficients such that $\sum_{k=1}^{\infty}(|a_k| + |b_k|) < \infty$, then the Fourier series of f converges uniformly to it.

Problem 4

Show that the Fourier series for $f(x) = x^2$ on $[-\pi, \pi]$ is given by

$$\frac{\pi^2}{3} - 4\sum_{k=1}^{\infty}(-1)^{k-1}\frac{\cos(kx)}{k^2}$$

and using the result on (uniform) convergence together with an appropriate choice of x, show

$$\sum_{k=1}^{\infty}\frac{1}{k^2} = \frac{\pi^2}{6}\,.$$