MATH 4332/6313

Introduction to Real Analysis

Spring 2018

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 1, due Thursday, February 1, 8:30am

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class or in the preceding term in support of your reasoning.

Problem 1

Based on the definition of open sets in the metric space $\mathbb{R}^2$ equipped with the Euclidean metric, prove that the set $A = \{(x_1, x_2) : x_1 + x_2 > 0\}$ is open.

Problem 2

Show that every open set A in a metric space (X, d) is the union of closed sets.

Problem 3

Let $X = C([0,1])$ be the space of continuous real-valued functions on $[0,1]$ with the max-metric

$$d_\infty(f, g) = \max\{|f(t) - g(t)| : 0 \le t \le 1\}\,.$$

Prove that the set $P = \{f \in C([0,1]) : f(t) > 0 \text{ for all } 0 \le t \le 1\}$ is open.

Problem 4

Let (X, d) be a metric space and $A \subset X$. Let E be the set of all $p \in X$ for which there is a sequence $\{p_n\}_{n\in\mathbb{N}}$ with $p_n \in A$ for each $n \in \mathbb{N}$ and $\lim_{n\to\infty} p_n = p$. Show that E is the closure of A.

Problem 5

Show that a metric space (X, d) is complete if and only if every nested decreasing sequence of closed balls $\overline{B}_{r_j}(x_j)$ with radii $r_j \to 0$ has a non-empty intersection $\cap_{j=1}^\infty \overline{B}_{r_j}(x_j)$.