Department of Mathematics University of Houston

MATH 4332/6313

Introduction to Real Analysis

Spring 2018

First name: ____________________ **Last name:** ____________________ **Points:**

Assignment 2, due Thursday, February 8, 8:30am

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Let $\mathbb{R}$ be equipped with the usual metric and $A = \{\frac{1}{n} : n \in \mathbb{N}\}$.

a. Show that A is not compact.

b. Show that $A \cup \{0\}$ is compact without appealing to the characterization of compactness in Euclidean spaces.

Problem 2

Let (X, d) be a metric space and K_1, K_2, $\ldots$, K_n be compact subsets of X. Prove that $K = K_1 \cup K_2 \cup \ldots K_n$ is compact.

Problem 3

Let (X, d) be a metric space and $K \subset X$ be compact. Prove that K is bounded.

Problem 4

Let (X, d) be a metric space, $Y \subset X$ and consider the metric space (Y, d).

a. Show that every open set U in Y has the form $U = V \cap Y$ for an open set V in X. Hint: Show this first for open balls.

b. Show that Y is compact in (Y, d) if and only if every collection of open sets $\{V_j\}_{j \in J}$ in X that covers Y has a finite subcover.