Department of Mathematics University of Houston

MATH 4332/6313
Introduction to Real Analysis
Spring 2018

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 5, due Thursday, March 8, 8:30am

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Let (X, d), (Y, ρ) and (Z, σ) be metric spaces and $f : X \to Y$ be a contraction with Lipschitz constant $r < 1$, $g : Y \to Z$ a contraction with Lipschitz constant $s < 1$. Prove that the composition $h = g \circ f : X \to Z$ has Lipschitz constant rs.

Problem 2

Show that $f(x) = \sin(x)$ is not a contraction on $[-1, 1]$.

Problem 3

Let $f(x) = \frac{x}{2} + \frac{1}{x}$. Use some basic calculus to show that f maps $[1, 2]$ into $[1, 2]$, and use the mean value theorem to show that it is a contraction mapping. What is the value of the unique fixed point x^*? If you choose $x_1 = \frac{3}{2}$ as your starting value, estimate $|x^* - x_n|$ for $n \in \mathbb{N}$.

Problem 4

Let $f(x) = \frac{x}{2} - 3$. Starting from $x_1 = 1$, compute the explicit value of x_n if we let $x_n = f(x_{n-1})$. Find the limit $x^* = \lim_{n\to\infty} x_n$ and verify $f(x^*) = x^*$.