Department of Mathematics University of Houston

MATH 4332/6313

Introduction to Real Analysis

Spring 2018

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 7, due Thursday, March 29, 8:30am

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Find the Taylor polynomials of order 3 at the point a for the choices of the function f given below and estimate/bound the error of approximating f by its Taylor polynomial at the point b.

a. $f(x) = \tan x$, $a = \pi/4$, $b = 3/4$;

b. $f(x) = \sqrt{1 + x^2}$, $a = 0$, $b = 0.1$;

c. $f(x) = x^4$, $a = 1$, $b = 0.99$.

Problem 2

Use the convergence of the Taylor series of $\sin(x)$ about $x = \pi/6$ to approximate $\sin(31°)$ to 10 decimal places.

Problem 3

Show that if f is a continuous function on $[0, 1]$ such that $\int_0^1 f(x)x^n dx = 0$ for each integer $n \geq 0$, then $f \equiv 0$. Hint: Recall that if $(g_n)_{n=1}^\infty$ is a uniformly convergent sequence in $C([0, 1])$ with limit g, then $\lim_{n\to\infty} \int_0^1 g_n(x)dx = \int_0^1 g(x)dx$. Use this idea to first prove $\int_0^1 |f(x)|^2 dx = 0$ and then deduce $f \equiv 0$.

Problem 4

Suppose $f \in C([0, 1])$ satisfies $f(0) = f(1) = 0$. Show that f is the (uniform) limit of a sequence of polynomials $(p_n)_{n=1}^\infty$ with $p_n(0) = p_n(1) = 0$ for each $n \in \mathbb{N}$.