Practice Exam 1 – Math 4332/6313
February, 2018

First name: ________________ Last name: ________________ Last 4 digits student ID: ______

1 True-False Problems (4 points each)

Put a circle around T beside each statement that is true, and a circle around F beside each statement that is false.

Throughout (X, d) denotes an arbitrary metric space.

T / F For $r > 0, p \in X$ the set $B_r(p)$ is never a closed set.

T / F If $C \subseteq X$ has the property that every sequence has a subsequence that converges to a point in C, then C is closed.

T / F If $f : X \to Y$ is a function, (X, d) is the discrete metric space and (Y, ρ) is any metric space then f is continuous.

T / F If $K \subseteq X$ has the property that every convergent sequence in K is bounded, then K is compact.

T / F If C is closed and U is open, then $C \cup U'$ is closed.

T / F A closed and bounded subset of a metric space is compact.

In the following problems, you may quote statements from class or homework to simplify your answers.

2 Problem

Let $X = [0,1) \cup (2,3]$, with metric d given by $d(x,y) = |x-y|$. Prove that $[0,1)$ is a closed subset of X.

3 Problem

Let (X, d) be a metric space and $A \subset X$ be totally bounded. Show that the closure $\overline{A}$ is totally bounded.

4 Problem

Let ρ be the discrete metric on $\mathbb{R}$ and d be the usual metric on $\mathbb{R}$. Show that the function $f(x) = x$ from $(\mathbb{R}, \rho)$ to $(\mathbb{R}, d)$ is continuous and one-to-one but that f^{-1} is not continuous.

5 Problem

Let (K, d) be a compact metric space and f_1, f_2 (uniformly) continuous, real-valued functions on K. Prove that $f_1 f_2 : x \mapsto f_1(x) f_2(x)$ is uniformly continuous on K.

6 Problem

Let (X, d) be a metric space and let $K_1,\ K_2,\ K_3, \ldots$ be a sequence of nonempty compact sets in X with $K_1 \supset K_2 \supset \cdots \supset K_j \supset K_{j+1}$ for any $j \geq 2$, then show that $\cap_{j=1}^{\infty} K_j$ is non-empty. Hint: Consider a sequence $\{x_n\}_{n=1}^{\infty}$ with $x_n \in K_n$ for each $n \in \mathbb{N}$.

[empty page]