Math 4332/6313

Introduction to Real Analysis

Spring 2020

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 3, due Thursday, February 6, 8:30am

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class or in the preceding term in support of your reasoning.

Problem 1

Show that every open set A in a metric space (X, d) is the union of closed sets.

Problem 2

Let $X = C([0, 1])$ be the space of continuous real-valued functions on $[0, 1]$ with the max-metric

$$d_\infty(f, g) = \max\{|f(t) - g(t)| : 0 \leq t \leq 1\}\,.$$

Prove that the set $P = \{f \in C([0, 1]) : f(t) \geq 0 \text{ for all } 0 \leq t \leq 1\}$ is closed.

Problem 3

Let (X, d) be a metric space and $A \subset X$. The closure $\overline{A}$ of the set A is defined as the set of all $p \in X$ for which there is a sequence $(p_n)_{n\in\mathbb{N}}$ with $p_n \in A$ for each $n \in \mathbb{N}$ and $\lim_{n\to\infty} p_n = p$. Show that $\overline{A}$ is closed and show that if C is a closed set with $A \subset C$, then $\overline{A} \subset C$. Informally, this could be stated as "$\overline{A}$ is the smallest closed set containing A".

Problem 4

Let (X, d) be a metric space, $Y \subset X$ and consider the metric space (Y, d). Show that every open set U in Y has the form $U = V \cap Y$ for an open set V in X. Hint: Show this first for open balls.