Department of Mathematics University of Houston

MATH 4332/6313

Introduction to Real Analysis

Spring 2020

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 4, due Thursday, February 13, 8:30am

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Let (X, d) be a metric space and $K \subset X$ be compact. Prove that K is bounded.

Problem 2

Let $\mathbb{R}$ be equipped with the usual metric and $A = \{\frac{1}{n} : n \in \mathbb{N}\}$.

a. Show that A is not compact.

b. Show that $A \cup \{0\}$ is compact.

Problem 3

Let (X, d) be a metric space and K_1, K_2, $\ldots$, K_n be compact subsets of X. Prove that $K = K_1 \cup K_2 \cup \ldots K_n$ is compact.

Problem 4

Let (K, d) be a compact metric space and let $C_0 = K$, $C_j \supset C_{j+1}$ for each $j \in \mathbb{N}$ define a nested sequence of closed, non-empty sets, then show $\bigcap_j C_j \neq \emptyset$. Hint: Use the finite subcover property of K in an indirect proof with $U_j = C_j'$.