Department of Mathematics University of Houston

MATH 4332/6313

Introduction to Real Analysis

Spring 2020

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 5, due Thursday, February 20, 8:30am

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Find an example for two metric spaces (X, d) and (Y, ρ), a continuous function $f : X \to Y$ and a Cauchy sequence $\{p_n\}_{n\in\mathbb{N}}$ in X which is not mapped to a Cauchy sequence in Y.

Problem 2

Let (X, d) and (Y, ρ) be metric spaces. Prove that if $f : X \to Y$ is continuous, then for any set A in X with closure $\overline{A}$,

a. we have $f(\overline{A}) \subset \overline{f(A)}$

b. and this inclusion can be proper, i.e. give an example for which $f(\overline{A}) \neq \overline{f(A)}$.

Problem 3

Let (X, d) and (Y, ρ) be metric spaces, X compact. Show that if $f : X \to Y$ is continuous, one-to-one and onto, then f^{-1} is continuous.

Problem 4

Let $X = [0, \infty)$, equipped with the usual metric from $\mathbb{R}$. Show that the function $f(x) = \sqrt{x}$ is uniformly continuous on X. Hint: Use the inequality $\sqrt{a + b} \leq \sqrt{a} + \sqrt{b}$ for any $a, b \geq 0$ (proved by squaring both sides).

Problem 5

Show that if (X, d) and (Y, ρ) are two metric spaces, X is compact and $f : X \to Y$ is continuous, then f is uniformly continuous.