

MATH 4332
Introduction to Real Analysis
Spring 2020

First name: _____ Last name: _____

Points:

Assignment 9, due Thursday, April 16, 8:30am

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Consider the ODE $y' = (1/5) \sin(xy)$ with initial value $y(0) = y_0$ and its solution f on the interval $[0, \pi/2]$. If you only experimentally measure the initial value y_0 and obtain a solution g of this ODE with initial value $y_0 + \delta$ instead of y_0 and want to achieve an accuracy of your solution $d_\infty(f, g) < \epsilon$, how small should δ be to guarantee this?

Problem 2

Let g and h be real-valued functions on \mathbb{R} such that g is differentiable at a and h is differentiable at b . Show that $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ defined by $f(x_1, x_2) = g(x_1)h(x_2)$ is differentiable at $x_0 = (a, b)$.

Problem 3

Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ be given by $f(x) = \frac{\|x\|^4}{1+\|x\|^2}$. Use the chain rule to show that f is differentiable at each $x \in \mathbb{R}^n$ and compute $Df(x)$.

Problem 4

Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, $f(x, y) = (x^2 - y^2, 2xy)$. For which $(x, y) \in \mathbb{R}^2$ is there a ball $B_\epsilon(x, y)$ with some $\epsilon > 0$ so that f restricted to this ball has an inverse? Explain your answer by referring to a result from the notes.