Practice Exam 1 – Math 4332/6313
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1 True-False Problems

Put a circle around T beside each statement that is true, and a circle around F beside each statement that is false.

Throughout (X, d) denotes an arbitrary metric space.

T / F For $r > 0, p \in X$ the set $B_r(p)$ is never a closed set.

T / F If $C \subseteq X$ has the property that every sequence has a subsequence that converges to a point in C, then C is closed.

T / F If $f : X \to Y$ is a function, (X, d) is the discrete metric space and (Y, ρ) is any metric space then f is continuous.

T / F If $K \subseteq X$ has the property that every convergent sequence in K is bounded, then K is compact.

T / F If C is closed and U is open, then $C \cup U'$ is closed.

T / F A closed and bounded subset of a metric space is compact.

In the following problems, you may quote statements from class or homework to simplify your answers.

2 Problem

Let (X, d) be a metric space and K_1, K_2 compact subsets of X. Prove that $K = K_1 \cap K_2$ is compact.

3 Problem

Let $X = C([0,1])$, equipped with the metric d_∞, and $K \subset X$ given by

$$K = \{f : [0,t] \to \mathbb{R}, f(0) = 0 \text{ and } f \text{ is } L\text{-Lipschitz continuous with constant } L \leq 1\}.$$

Explain why K is compact.

4 Problem

Let (X, d) be a metric space and $A \subset X$ be totally bounded. Show that the closure $\overline{A}$ is totally bounded.

5 Problem

Let ρ be the discrete metric on $\mathbb{R}$ and d be the usual metric on $\mathbb{R}$. Show that the function $f(x) = x$ from $(\mathbb{R}, \rho)$ to $(\mathbb{R}, d)$ is continuous and one-to-one but that the inverse mapping f^{-1} is not continuous.

6 Problem

Let (K, d) be a compact metric space and f_1, f_2 (uniformly) continuous, real-valued functions on K. Prove that $f_1 f_2 : x \mapsto f_1(x) f_2(x)$ is uniformly continuous on K.

7 Problem

Prove that if $\mathbb{R}$ is equipped with the usual metric d, then $Y = [0, 1]$ and d form the complete metric space $([0, 1], d)$.

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