Math 4397/6397, Fall 2009
Problem Set 1, due Thursday, Sep 3

Problem 1. Show the following with the help of the axioms for probability measures. You may state and use set-theoretic identities without further explanation.

a. $P(\emptyset) = 0$.

b. If $A \subset B$ then $P(A) \leq P(B)$.

c. For any A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

d. $P(A \cap B^c) = P(A) - P(A \cap B)$.

e. $P(\cup_{i=1}^n E_i) \geq \max_i P(E_i)$.

Problem 2. Suppose that an influenza epidemic strikes a city. In 17% of two parent families at least one of the parents has contracted the disease. In 12% of the families the father has contracted influenza while in 6% of the families both the mother and father have contracted influenza.

a. Compute the probability that the mother has contracted influenza.

b. Compute the probability that neither the mother nor the father has contracted influenza.

c. Compute the probability that the mother has contracted influenza but the father has not.

d. Compute the probability that the father has contracted influenza but the mother has not.

Problem 3. The logistic density is defined by

$$f(x) = \frac{e^{-x}}{(1+e^{-x})^2} \qquad \text{for } -\infty < x < \infty.$$

a. Show that this is a valid density.

b. Calculate the cumulative distribution function associated with this density.

c. What value do you get when you plug 0 into the distribution function? If X is a random variable with this distribution function, interpret what this result means for X.

d. Define the *odds* of an event with probability p as $p/(1-p)$. Prove that the p^{th} quantile from this distribution is $\log\{p/(1-p)\}$; which is the natural log of the odds of an event with probability p.