Math 4397/6397, Fall 2009
Problem Set 2, due Thursday, Sep 10

Problem 1. Using the rules of expectations prove that $\mathrm{Var}(X) = E[X^2] - E[X]^2$ starting from the definition $\mathrm{Var}(X) = E[(X-\mu)^2]$.

Problem 2. Suppose that a random variable X has a density of the form

$$f(x) = \begin{cases} cx^k & , 0 \le x \le 1 \\ 0 & , \text{ else} \end{cases}$$

for some constant $k \ge 0$.

a. Find c.

b. Derive the distribution function for the density f.

c. Derive a formula for the p^{th} quantile from f.

d. Let $0 \le a < b \le 1$. Compute $P(a < X < b)$.

e. Compute the mean and the variance of X.

Problem 3. Let $g(x) = sf_1(x) + (1-s)f_2(x)$ where $0 \le s \le 1$ and the densities f_1 and f_2 are associated with means and variances μ_1, σ_1^2 and μ_2, σ_2^2, respectively. Show that g is a valid density, and compute its associated mean and variance.

Problem 4. You are playing a game with a friend where you flip a coin and if it comes up heads you give him a dollar and if it comes up tails she gives you a dollar. You play the game ten times.

a. What is the expected total earnings for you? (Show your work; state your assumptions.)

b. What is the variance of your total earnings? (Show your work; state your assumptions.)

c. Suppose that the die is biased and you have a $.4$ chance of winning for each flip. repeat the calculations in parts a and b

Problem 5. Assume that swimming in the bayou is associated with $1/500$ probability of getting an infection. How many times would an uninfected person have to have repeat a swim in the bayou to have a 50% probability of obtaining an infection? State the assumptions of your calculations.