

MATH 6360

Applicable Analysis

- Class:** TuTh 11:30am-12:50pm, S 116 (Science Building 116)
- Instructor:** Bernhard Bodmann, *bgb@math.uh.edu*
- Office:** Tu 1-2pm, We 1-2pm (online, on MS Teams)
- Topics:** This course covers topics in analysis that are motivated by applications.

Review of metric spaces, completeness, characterization of compactness.

Contraction mappings and fixed points. Applications of contractions mappings: integral equations, solutions to initial value problems. Local existence and uniqueness of solutions, stability.

L^p spaces as metric completions. Extending the Riemann integral to L^p spaces. Banach spaces.

Dual spaces. Uniform boundedness.

Consequences of uniform boundedness for Fourier series and polynomial interpolation.

Uniform convexity, best approximation property and duality for L^p -spaces.

Bounded inverse, closed graph theorem.

Hilbert spaces. Orthonormal bases and their characterization. Characterization of best approximation by orthogonal projection. Fourier series. Convergence in L^2 and pointwise convergence. Weak convergence. Nonlinear best approximations and (approximate) sparsity.

Topics covered in the sequel, Math 6361, include: Relationships between weak and norm convergence. Weak compactness in Hilbert spaces. Linear and convex programming in Hilbert spaces.

Operators and bilinear forms. The Lax-Milgram theorem.

Linear inverse problems. Sparse recovery by norm minimization.

The Hilbert-Schmidt norm and Hilbert-Schmidt operators. Compact self-adjoint operators. The spectral theorem for compact, self-adjoint operators. Diagonalizing normal operators. Solutions to Schrödinger's eigenvalue problem and compact integral operators.

- Text:** No obligatory text.

Part of the material will come from K. Davidson and A. Donsig, *Real Analysis with Applications: Theory in Practice*, Springer, 2014. The second term contains more Functional Analysis content, as covered in D. Werner, *Functional Analysis*, Springer, 2005, or E. Kreyszig, *Introductory Functional Analysis with Applications*, Wiley, 1989. Some very recent topics will be worked out based on S. Foucart and H. Rauhut, *A Mathematical Introduction to Compressed Sensing*, Birkhäuser, 2013.

- Assignments:** There will be approximately 10 homework sets.

Exams, Grades: The midterms are in class, Sept 30 and Nov 18, the final as scheduled by the registrar. The midterms count 20% each, homework and final contribute 30% to your grade.

Disabilities: If there are aspects of the instruction or design of this course that result in barriers to your inclusion or accurate assessment or achievement, please notify the instructor as soon as possible. Students are also welcome to contact the Center for Students with DisABILITIES at 713-743-5400 or uhcsd@central.uh.edu to discuss a range of options to removing barriers in the course, including reasonable academic adjustments/auxiliary aids in accordance with the Section 504 of the Rehabilitation Act of 1973 and the Americans with Disabilities Act of 1990 guidelines.

Counseling: Counseling and Psychological Services (CAPS) can help students who are having difficulties managing stress, adjusting to college, or feeling sad and hopeless. You can reach CAPS (www.uh.edu/caps) by calling 713-743-5454 during and after business hours for routine appointments or if you or someone you know is in crisis. No appointment is necessary for the “Let’s Talk” program, a drop-in consultation service at convenient locations and hours around campus. http://www.uh.edu/caps/outreach/lets_talk.html

Course Policy: Students are expected to know what constitutes academic integrity. They are expected to avoid committing academic offenses, and to take responsibility for their actions. Students who are unsure whether an action constitutes an offense, or who need help in learning how to avoid offenses (e.g., plagiarism or cheating) or who would like to learn about rules for group work should seek guidance from the course instructor, teaching assistant, academic advisor, or the Undergraduate Associate Dean. Students who feel that they have been wrongfully or unjustly penalized have the right to grieve.

CoViD Mitigation: To reduce the spread of COVID-19, the University strongly encourages everyone (vaccinated or not) to wear face coverings indoors on campus including classrooms for both faculty and students.

Your presence in class each session means that you:

- Are NOT exhibiting any Coronavirus Symptoms that makes you think that you may have COVID-19
- Have NOT tested positive or been diagnosed for COVID-19
- Have NOT knowingly been exposed to someone with COVID-19 or suspected/presumed COVID-19

If you are experiencing any COVID-19 symptoms that are not clearly related to a pre-existing medical condition, do not come to class. Please see Student Protocols

<https://www.uh.edu/covid-19/guidelines-protocols/diagnosis-symptoms/#students>

for what to do if you experience symptoms and Potential Exposure to Coronavirus for what to do if you have potentially been exposed to COVID-19. Consult the Graduate Excused Absence Policy for information regarding excused absences due to medical reasons, see

<https://uh.edu/provost/policies-resources/student/excused-absence-policy>