MATH 6360

Applied Analysis

Fall 2018

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 6, due Friday, October 19, 10am

Please staple this problem sheet to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Let $C([-1,1])$ be equipped with the norm $\|f\|_1 = \int_{-1}^1 |f(x)|dx$. Does the linear functional $F : f \mapsto f(0)$ extend uniquely to $L^1([-1,1])$?

Problem 2

Show that if X and Y are normed spaces, and $B(X,Y)$ is a Banach space, then Y is a Banach space. Hint: If $F \in X^*$ is not the zero functional, consider $T : Y \to B(X,Y)$ given by $T(y)x = F(x)y$.

Problem 3

Let c_0 be the normed vector space containing each sequence $x = (x_n)_{n\in\mathbb{N}}$ with $\lim_n x_n = 0$, equipped with the norm $\|x\|_\infty = \sup_n |x_n|$ for $x \in c_0$. Show that the dual space c_0^* is isometrically isomorphic to ℓ^1, the space of summable sequences, equipped with $\|y\|_1 = \sum_{n=1}^\infty |y_n|$ for $y \in \ell^1$.

Problem 4

Let $\mathrm{Pol}([0,1])$ be the space of real polynomials on $[0,1]$, equipped with $\|p\|_1 = \int_0^1 |p(x)|dx$. Define for $p \in \mathrm{Pol}([0,1])$ the map $T(p)q = \int_0^1 p(x)q(x)dx$, then show $T(p)$ is continuous, so $T : \mathrm{Pol}([0,1]) \to \mathrm{Pol}([0,1])^*$. Moreover, show that if $A = \{p \in \mathrm{Pol}([0,1]) : \|p\|_1 \le 1\}$, then for each $q \in \mathrm{Pol}([0,1])$, $\sup_{p\in A} |T(p)q|$ is finite, but T is not uniformly bounded. Why does this not contradict the Banach-Steinhaus theorem?