Practice Exam 1 – Math 6360
September, 2018

First name: ____________________ Last name: ____________________ Last 4 SID: _____________

1 Memorization

1. State the definition of a contraction mapping T on a metric space (X, d).

2. State the theorem on local invertibility of a multivariate function $f : \mathbb{E} \to \mathbb{R}^n$.

In the following problems, you may quote statements from class to simplify your answers. You do not need to give a proof of a statement if it was discussed in class.

2 Problem

Consider $E = \{f \in C([0,1]) : 0 \leq f(x) \leq 1 \text{ for each } x \in [0,1]\}$. Let $\alpha \in (0,1)$ and the map T be defined by

$$Tf(x) = \frac{x}{3} + \alpha \int_0^1 xt(f(t))^2 dt$$

1. Prove that T is a contraction mapping on E.

2. Determine a fixed point f^* of T.

3 Problem

Let $h : \mathbb{R}^3 \to \mathbb{R}$ be given by $h(x, y, z) = x^2y^2z^2$.

1. Determine the points at which h assumes its maximum on the sphere $S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$ and the extreme values of h.

2. Use this result to show that for $a, b, c \geq 0$, $(abc)^{1/3} \leq \frac{a+b+c}{3}$. Hint: If a, b or $c > 0$, let $x = \sqrt{a/(a+b+c)}$, $y = \sqrt{b/(a+b+c)}$ and $z = \sqrt{c/(a+b+c)}$.

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