Practice Exam 2 – Math 6360
November, 2018

First name: ____________________ Last name: ____________________ Last 4 SID: _____________

1 Memorization

1. State a theorem on uniform boundedness of $(T_a)_{a\in A}$ where each $T_a : X \to Y$ with appropriate spaces X and Y.

2. State a best approximation property in $L^p([a,b])$.

In the following problems, you may quote statements from class to simplify your answers. You do not need to give a proof of a statement if it was discussed in class.

2 Problem

Let X and Y be Banach spaces and S a dense subset of X. Let $T' : S \to Y$ be a bounded linear map, then show that there is a unique bounded, linear map $T : X \to Y$ such that $T(x) = T'(x)$ for $x \in S$.

3 Problem

Recall that the space ℓ^p is usually equipped with the p-norm that assigns to $x = (x_k)_{k=1}^{\infty} \in \ell^p$ the norm $\|x\|_p = (\sum_k |x_k|^p)^{1/p}$. Consider $X = \ell^1$ as a vector space. From $\ell^1 \subset \ell^2$ we can also put the 2-norm on X. Is there a $C \geq 1$ such that for each $x \in X$, $\|x\|_2/C \leq \|x\|_1 \leq C\|x\|_2$? Explain the reason for your answer. Hint: Is X a closed subspace of ℓ^2?

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