Practice Exam 1 – Math 6361
February, 2018

First name: ____________________ Last name: ____________________ Last 4 SID: _____________

1 Memorization

1. State the parallelogram identity for vectors in Hilbert spaces. What is its relevance for determining when a norm is induced by an inner product?

2. State the Lax-Milgram theorem for a sesquilinear form on a complex Hilbert space.

In the following problems, you may quote statements from class to simplify your answers. You do not need to give a proof of a statement if it was discussed in class.

2 Problem

Let the vectors $(u_n)_{n=-\infty}^{\infty}$ in $L^2([-\pi,\pi])$ be given by $u_n = \frac{1}{\sqrt{2\pi}}e^{inx}$. Let $f \in C([-\pi,\pi])$ and define $g(x) = \int_0^x f(t)dt$.

1. If $f(x) = u_n(x)$, $n \in \mathbb{Z} \setminus \{0\}$, find $g(x)$.

2. If $f \in C([-\pi,\pi])$ and $f = \sum_{n\neq 0} c_n u_n$ with $c_n \in \mathbb{C}$ for $n \in \mathbb{Z}$, find $d_n = \langle g, u_n\rangle$ for each $n \in \mathbb{Z}$. What can you say about the convergence of the Fourier series of g?

3 Problem

Let $x^{(n)}$ be a sequence in ℓ^2 defined by $x_k^{(n)} = \frac{1}{\sqrt{n}}$ if $k \leq n$ and $x_k^{(n)} = 0$ if $k > n$. Show that $x^{(n)}$ converges weakly to 0, but is not norm-convergent.

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