HW #3. Due *Friday, March 31.*

1. *(Katok–Hasselblatt 3.2.2–3.)* Let $f\colon S^1 \to S^1$ be a continuous map.
(a) Prove that $h(f) \geq \log|\deg f|$.
(b) Prove that if in addition f is monotone (that is, its lift to $\mathbb{R}$ is a monotone function), then $h(f) = \log|\deg f|$.

2. *(Katok–Hasselblatt 4.1.2.)* Show that different ergodic probability measures are mutually singular. That is, show that if μ, ν are ergodic probability measures with $\mu \neq \nu$, then there are disjoint sets A, B such that $\mu(A) = 1$ and $\nu(B) = 1$.

3. *(Katok–Hasselblatt 4.1.11.)* Prove that a measure-preserving transformation (X, T, μ) is ergodic if and only if 1 is a simple eigenvalue of the Koopman operator $U_T\colon L^2(\mu) \to L^2(\mu)$. (Recall that $(U_T\phi)(x) = \phi(Tx)$.)

4. *(Katok–Hasselblatt 4.2.16.)* Prove that for the Markov measure μ_Π, where Π is a primitive stochastic matrix, mixing is exponential on cylinders, that is, for any cylinders C, C' we have

$$|\mu_\Pi(\sigma^{-n}(C) \cap C') - \mu_\Pi(C) \cdot \mu_\Pi(C')| < ce^{-\alpha n}$$

for some $c, \alpha > 0$.