Abstracts – Invited Talks

Speaker: **Keith Burns** (Northwestern)

Title: Measures of maximal entropy for the geodesic flows of surfaces with caps – part 1

Abstract: We prove that the geodesic flow on a compact surface with negative curvature in the complement of a disjoint union of symmetric caps with monotone curvature has a unique measure of maximal entropy. In this first talk we will discuss the class of surfaces we are interested in, the geometry of caps, and how geodesics behave when entering and exiting a cap. We will then outline the proof of the main result. This is joint work with Todd Fisher and Rachel McEnroe.

Speaker: **Mark Demers** (Fairfield)

Title: Thermodynamic formalism for finite horizon dispersing billiards

Abstract: Mathematical billiards are foundational models from statistical mechanics in which point particles collide elastically with fixed boundaries. For a class of finite horizon dispersing billiards, we review recent results proving the existence and uniqueness of equilibrium states for a family of geometric potentials, $-t \log J^u T$, $t \geq 0$. The importance of this family stems from the fact that $t = 1$ corresponds to the smooth invariant (SRB) measure, while $t = 0$ corresponds to the measure of maximal entropy. By constructing anisotropic Banach spaces adapted to the potentials, we are able to prove exponential mixing by way of a spectral gap for the associated transfer operator. Yet the spectral gap vanishes as $t \to 0$ and we discuss a possible phase transition for the billiard at $t = 0$.

Speaker: **Alena Erchenko** (Stony Brook)

Title: Flexibility and rigidity questions for Cantor repellers

Abstract: We will consider dynamical systems that we call Cantor repellers which are expanding maps on invariant Cantor sets coming from iterated function systems. Cantor repellers have two natural invariant measures: the measure of full dimension and the measure of maximal entropy. We show that dimensions and Lyapunov exponents of those measures are flexible up to well understood restrictions. We will also describe a rigidity result that corresponds to the boundary case for the range of values of the considered dynamical data. This is joint work with Jacob Mazor.

Speaker: **Todd Fisher** (Brigham Young)

Title: Measures of maximal entropy for the geodesic flows of surfaces with caps – part 2

Abstract: We prove that the geodesic flow on a compact surface with negative curvature in the complement of a disjoint union of symmetric caps with monotone curvature has a unique measure of maximal entropy. In this second talk we will review the program of Climenhaga and Thompson and explain how to obtain the necessary hypotheses for the geodesic flow on these surfaces. We also explain a modified flow and how this is needed in obtaining our results. This is joint work with Keith Burns and Rachel McEnroe.

Speaker: **Yan Mary He** (Oklahoma)

Title: Complex dynamics, big mapping class groups and Nielsen realization

Abstract: In this talk, we will first see how surfaces of infinite type and their mapping class groups naturally arise in dynamics, especially in complex dynamics. Then we show that the mapping class group of the plane minus a Cantor set or the sphere minus a Cantor set cannot be realized as a subgroup of the homeomorphism group. This is joint work with Lei Chen.

Speaker: **Lien-Yung "Nyima" Kao** (George Washington)

Title: Pressure metrics on deformation spaces

Abstract: In this talk, we will discuss some basic ideas of a dynamically defined Riemannian metric on deformation spaces – the pressure metric. It has rich applications in geometry. For example, the pressure metric is, indeed, the famous Weil-Petersson metric on Teichmüller spaces. We will start from a more dynamic originated deformation space, the moduli space of metric graphs, to more geometric originated deformation spaces such as (cusped) Teichmüller space and Hitchin components. The new results in this talk are based on joint works with Harry Bray, Dick Canary, and Giuseppe Martone.

Speaker: **Kathryn Lindsey** (Boston College)

Title: Entropy and uniformly expanding models of 1D dynamical systems

Abstract: Kneading theory shows that every continuous, multimodal self-map of a real interval is semiconjugate to a piecewise linear, "uniformly expanding" interval self-map with the same topological entropy. The situation for polynomials in one complex variable with connected Julia set is similar – the Hubbard tree admits a topological model (a self-map of a tree) on which the dynamics are uniformly expanding. When the map (real or complex) is postcritically finite, the exponential of this topological entropy is the largest eigenvalue of the matrix associated to the Markov partition of the Hubbard tree obtained by cutting the tree at the postcritical set. G. Tiozzo, C. Wu and I investigated how the eigenvalues of this Markov matrix associated to rational external angles of the Mandelbrot set vary with the angle. We defined "Master Teapots" associated to principal veins in the Mandelbrot set and proved that the eigenvalues outside the unit circle move continuously while roots inside the unit circle "persist." This talk will discuss this circle of ideas and results, and is based on joint work with G. Tiozzo and C. Wu.

Speaker: **Matt Nicol** (Houston)

Title: Stable laws for random dynamical systems

Abstract: Joint with Romain Aimino and Andrew Török. We consider random dynamical systems formed by concatenating maps acting on the unit interval $[0, 1]$ in an iid fashion. Considered as a stationary Markov process, the random dynamical system possesses a unique stationary measure ν. We consider a class of non square-integrable observables ϕ, mostly of form $\phi(x) = d(x, x_0)^{-\frac{1}{\alpha}}$ where x_0 is a non-periodic point satisfying some other genericity conditions, and more generally regularly varying observables with index $\alpha \in (0, 2)$. The two types of maps we concatenate are a class of piecewise C^2 expanding maps, and a class of intermittent maps possessing an indifferent fixed point at the origin. Under conditions on the dynamics and α we establish Poisson limit laws, convergence of scaled Birkhoff sums to a stable limit law and functional stable limit laws, in both the annealed and quenched case.

Speaker: **Victoria Sadovskaya** (Penn State)

Title: Local rigidity for hyperbolic toral automorphisms

Abstract: We consider a hyperbolic toral automorphism L and its C^1-small perturbation f. It is well-known that f is Anosov and topologically conjugate to L, but a conjugacy H is only Hölder continuous in general. We discuss conditions for C^1 and C^∞ smoothness of H, in particular, conjugacy of the periodic data of f and L, coincidence of their Lyapunov exponents, and weak differentiability of H. Linear cocycles over hyperbolic systems play an important role in the arguments. The talk is based on joint works with A. Gogolev, B. Kalinin, and Z. Wang.

Speaker: **Fan Yang** (Michigan State)

Title: Uniqueness of equilibrium states for Lorenz attractors in any dimension

Abstract: Since its discovery in the 1960s, Lorenz attractor and Lorenz-like classes have played a central role in the study of continuous-time dynamical systems. However, there are very few results in dimensions higher than three due to the absence of a proper geometric model. In this talk, I will present some recent progress on the statistical properties of Lorenz attractors beyond dimension three, including expansivity, entropy estimate, and the existence of physical measures. I will also discuss recent work, joint with Jiagang Yang and Maria Jose Pacifico, that establishes the uniqueness of equilibrium states for "most" Hölder continuous functions, particularly the uniqueness of the measure of maximal entropy.

Speaker: **Agnieszka Zelerowicz** (Maryland)

Title: Lorentz gases on quasicrystals

Abstract: The Lorentz gas was originally introduced as a model for the movement of electrons in metals. It consists of a massless point particle (electron) moving through Euclidean space bouncing off a given set of scatterers $\mathcal{S}$ (atoms of the metal) with elastic collisions at the boundaries $\partial\mathcal{S}$. If the set of scatterers is periodic in space, then the quotient system, which is compact, is known as the Sinai billiard. There is a great body of work devoted to Sinai billiards and in many ways their dynamics is well understood. In contrast, very little is known about the behavior of the Lorentz gases with aperiodic configurations of scatterers which model quasicrystals and other low-complexity aperiodic sets. This case is the focus of our joint work with Rodrigo Treviño. We establish some dynamical properties which are common for the periodic and quasiperiodic billiards. We also point out some significant differences between the two. The novelty of our approach is the use of tiling spaces to obtain a compact model of the aperiodic Lorentz gas on the plane.

Speaker: **Hongkun Zhang** (Massachussetts–Amherst)

Title: Hyperbolicity of chaotic billiards

Abstract: Defocusing mechanism provides a way to construct chaotic (hyperbolic) billiards with focusing components by separating all regular components of the boundary of a billiard table sufficiently far away from each focusing component. If all focusing components of the boundary of the billiard table are circular arcs, then the above separation requirement reduces to that all circles obtained by completion of focusing components are contained in the billiard table. In this talk, we discuss a few classes of convex billiards, whose boundary consists of circular arcs. The hyperbolicity for some of these new billiards are numerically proved only, which leaves some open questions to explore.

Speaker: **Pengfei Zhang** (Oklahoma)

Title: Heteroclinic and homoclinic intersections for lemon billiards

Abstract: Let $\mathcal{Q}(b)$ be the planar domain obtained as the intersection of two unit disks, where $b \in (0, 2)$ measures the distance between their centers. The term *lemon billiards* has been chosen for the dynamical billiards on $\mathcal{Q}(b)$ since the domain $\mathcal{Q}(b)$ resembles the shape of a lemon. The lemon billiards have been studied by Heller and Tomsovic [2], in which they demonstrate a clear connection between the classical mechanics and the quantum mechanics: the eigenstates of the quantum billiards are large only at where the periodic trajectories of the classical billiards go. Numerical results in [1, Section IV] suggest that

Conjecture. *Let $F_b : M_b \to M_b$ be the billiard map on $\mathcal{Q}(b)$. Then $h_{\text{top}}(F_b) > 0$.*

In this talk I will show that for a range of parameters of the lemon billiards, there exist heteroclinic and homoclinic intersections for the billiard maps of the lemon billiards. In particular, such billiards have positive topological entropy. This is based on a joint work with Dr. Jin.

[1] J. Chen, L. Mohr, H.-K. Zhang, P. Zhang. *Ergodicity and coexistence of elliptic islands in a family of convex billiards.* Chaos **23** (2013), 043137.

[2] E. Heller and S. Tomsovic. *Postmodern quantum mechanics.* Physics Today **46** (1993), 38–46.

Abstracts – Grad Student Talks

Speaker: **Samuel Akingbade** (Yeshiva)

Title: Energy growth in Hamiltonian systems with small dissipation

Abstract: We consider a simple model of a mechanical system, consisting of a rotator and a pendulum with a small, periodic coupling, subject to a dissipative perturbation, where the dissipation coefficient is a fraction of the smallness parameter. The resulting system is non-Hamiltonian. We show that such a system exhibits Arnold diffusion, that is, there exist orbits for which the energy of the rotator subsystem grows by some quantity that is independent of the smallness parameter. For the unperturbed rotator-pendulum system, the energy of the rotator subsystem is conserved. The small, periodic coupling added to the system makes the rotator undergo small oscillations in energy, while the dissipative perturbation always yields a loss in energy. The physical significance of our result is that it is possible to overall gain a significant amount of energy over time. The Arnold diffusion phenomenon in systems with small dissipation was conjectured by Chirikov.

The methodology relies on the existence of a normally hyperbolic invariant manifold (NHIM) for the perturbed system. We use Melnikov theory to find conditions under which the stable and unstable manifolds of the NHIM intersect transversally, which allows us to define a scattering map on the NHIM. We compute the scattering map, and find conditions under which the scattering map increases the energy of the rotator. However, the dynamics restricted to the NHIM decreases the energy of the rotator. We show that, through a careful interplay between the two dynamics, we can nevertheless obtain trajectories along which the energy of the rotator overall increases. In this work, the rotator-pendulum coupling is carefully chosen, however we plan to show that the same mechanism works for general perturbations in future work.

A motivation for this work is furnished by energy harvesting devices. These consist of systems of oscillating beams made of piezoelectric materials, where on the one hand there is dissipation due to mechanical friction, and on the other hand there is external forcing that makes the beams oscillate.

Speaker: **Karen Butt** (Michigan)

Title: Quantitative marked length spectrum rigidity

Abstract: The marked length spectrum of a closed Riemannian manifold of negative curvature is a function on the free homotopy classes of closed curves which assigns to each class the length of its unique geodesic representative. Conjecturally, the marked length spectrum determines the metric up to isometry (Burns–Katok). This is known to be true in some special cases, namely in dimension 2 (Otal, Croke), in dimension at least 3 if one of the metrics is locally symmetric (Hamenstadt, Besson–Courtois–Gallot), and in any dimension if the metrics are assumed to be sufficiently close in a suitable C^k topology (Guillarmou–Knieper–Lefeuvre). Even in these cases, there is more to be

understood about to what extent the marked length spectrum determines the metric. Namely, if two manifolds have marked length spectra which are not equal but are close, is there some sense in which the metrics are close to being isometric? In this talk, we will provide some (quantitative) answers to this question, refining the known rigidity results for surfaces and for locally symmetric spaces of dimension at least 3.

Speaker: **Joondo Chang** (Memphis)

Title: Classical Hamiltonian dynamics and its applications to dynamical systems

Abstract: This talk will introduce the audience to Hamiltonian Dynamics and its associated physical properties in both classical and semi-classical physics. We shall discuss major classical theorems associated with Hamiltonian Physics, i.e., Poincaré Invariant Theorem, Liouville's Theorem, and Poisson's Theorem. Afterwards, we shall apply the major theorems to understand action-angle variables for smooth manifolds and the Arnold–Liouville Theorem on a d-dimensional Torus. If time permits, we shall also discuss about the application of Hamiltonian Dynamics to harmonic oscillators and classical pendulums.

Speaker: **Michael Chow** (Yale)

Title: Local mixing of one-parameter diagonal flows on Anosov homogeneous spaces

Abstract: Let G be a connected semisimple real algebraic group, $P < G$ be the minimal parabolic subgroup and $\Gamma < G$ be a Zariski dense P-Anosov subgroup. Anosov representations were first introduced by Labourie in the context of Hitchin representations for surface groups and further studied by Guichard and Wienhard who showed in particular that there is an abundance of Anosov subgroups. Let $A = \exp \mathfrak{a} < G$ be a maximal real split torus and $\mathfrak{a}^+ \subset \mathfrak{a}$ be a choice of positive Weyl chamber. Then a one-parameter diagonal flow on $\Gamma \backslash G$ is given by the right translation action by $\exp(tv)$, $t \in \mathbb{R}$ for some fixed $v \in \mathfrak{a}^+$. We prove that for v in the interior of the limit cone $\mathcal{L}_\Gamma$ introduced by Benoist, this diagonal flow is locally mixing with respect to the Bowen–Margulis–Sullivan measure. This mixing result has many applications to counting, equidistribution and measure classification as studied by many others – Margulis–Mohammadi–Oh, Sambarino, Edwards–Lee–Oh for example. Our work relies on obtaining a Markov section using the techniques of Bridgeman–Canary–Labourie–Sambarino and the theory of symbolic dynamics, thermodynamic formalism and transfer operators. Further work is near completion to upgrade to exponential mixing for Gromov geodesic flow and convex cocompact subgroups in rank 1. This is joint work with Pratyush Sarkar.

Speaker: **Spencer Durham** (Brigham Young)

Title: Minimality for iterated function systems of circle diffeomorphisms

Abstract: Barrientos and Raibekas showed that iterated function systems generated by two circle diffeomorphisms are minimal if and only if they satisfy certain conditions on their attracting periodic orbits and corresponding basins of attraction. We generalize to the case of finitely many generators and provide examples of what new behaviors can occur in this case. We also establish the robustness of minimality when it occurs and find an upper bound for the distortion from a rotation that generators can have with guaranteed minimality.

Speaker: **Greg Hemenway** (Houston)

Title: Equilibrium states for expanding skew products

Abstract: We will discuss the thermodynamic properties of equilibrium states for expanding skew products. We show how a family of fiberwise transfer operators can be used to define the conditional measures along fibers of the product. We prove that the push forward of the equilibrium state onto the base of the product is itself an equilibrium state for a Hölder potential defined via this fiberwise transfer operator.

Speaker: **Grigorii Monakov** (UC Irvine)

Title: Shadowing in linear skew products and large deviation estimates

Abstract: This talk is based on a research project under the supervision of Professor S. B. Tikhomirov. We investigate the probability of the event that a finite random pseudotrajectory can be effectively shadowed by an exact trajectory. The main result of the work describes a class of skew products, for which this probability tends to one as the length of a pseudotrajectory tends to infinity and the value of a maximal error on each step tends to zero. We also show that continuous linear skew products over a Bernoulli shift, doubling map on a circle and any Anosov linear map on a torus lie in this class. The Cramer's large deviation theorem is used in the proof.

Speaker: **William Wood** (UC Irvine)

Title: Uniform hyperbolicity and the periodic Anderson–Bernoulli model

Abstract: In this talk we will focus on the notion of uniform hyperbolicity of sets of matrices, and apply it to transfer matrices related to a discrete Schrödinger operator to study its spectrum. Specifically, we will show how to apply Johnson's Theorem that claims that a Schrödinger cocycle is uniformly hyperbolic if and only if the corresponding energy value is not in the spectrum, to the periodic Anderson–Bernoulli Model. As a result, we will prove that the spectrum of period two Anderson–Bernoulli Model consists of at most four intervals.
