UNIVERSITY OF HOUSTON
DEPARTMENT OF MATHEMATICS

Seminar on Partial Differential Equations

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Exterior Finite Energy Functions

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Abstract

In this talk, we introduce the *finite energy space* $E^{1,p}(U)$, where $U \subsetneq \mathbb{R}^N$ is an exterior region having compact, Lipschitz boundary ∂U, with $N \geq 3$ and $p \geq 1$. Functions in $E^{1,p}(U)$ are only required to *decay at infinity* in a broad measure-theoretic sense and be L^p-integrable of their weak gradients; moreover, mild locally integrable conditions are assumed.

When $1 < p < N$, $E^{1,p}(U)$ is a real Banach space under the gradient L^p-norm, and when $p = 2$, we denote $E^1(U) = E^{1,2}(U)$ which is a real Hilbert space with respect to the gradient L^2-inner product. Also, we have $W^{1,p}(U) \subsetneq E^{1,p}(U)$.

The harmonic Dirichlet-Poisson as well as Neumann and Robin problems are well-posed in $E^1(U)$. In addition, using the exterior harmonic Steklov eigenvalues and an associated family of eigenfunctions, spectral representation of solutions of these harmonic boundary value problems is given, and the exterior Poisson's kernel is described.

On the other hand, via these eigenvalues and eigenfunctions, a reproducing kernel of the harmonic subspace $\mathscr{H}(U)$ of $E^1(U)$ may be described explicitly.

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