UNIVERSITY OF HOUSTON
DEPARTMENT OF MATHEMATICS

Seminar on Partial Differential Equations

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Analysis of Reaction Diffusion Systems with Mass Transport Boundary Conditions

3:00 pm in 646 PGH
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Abstract

We are interested in reaction-diffusion systems in which some of the components react and diffuse in a smooth bounded 3 dimensional domain, and interact with other components that react and diffuse on the boundary of the domain. The simplest structure for a model of this type involves mass transport boundary conditions, and has the form:

$$\begin{aligned} u_t &= d_1 \Delta u + h(u), && \Omega \times (0,\infty) \\ \frac{\partial u}{\partial \eta} &= g(u,\nu), && \Gamma \times (0,\infty) \\ \nu_t &= d_2 \Delta_\Gamma \nu + f(u,\nu), && \Gamma \times (0,\infty) \\ u &= u_0 \text{ on } \Omega \times \{0\}, && \nu = \nu_0 \text{ on } \partial\Omega \times \{0\}. \end{aligned}$$

Here Ω is a smooth bounded domain in $\mathbb{R}^3$ with boundary Γ, $d_1,\ d_2 > 0$ are diffusion coefficients, η is the unit outward normal to Ω at each point on the boundary, Δ_Γ is the Laplace Beltrami operator, u_0 and ν_0 are bounded and nonnegative, and f, g, and h are smooth functions satisfying $f(z,0)$, $g(z,0)$, $h(0) \geq 0$ for all $z \geq 0$. Natural assumptions on f, g, and h lead to global existence and uniform bounds for solutions. Our analysis is applied to a mathematical model associated with the positioning of the FtsZ contractile ring by a min-dependent mechanism during the cell division process. Open mathematical questions are also discussed.

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