

# Contributed Talks

## CombinaTexas, Saturday-Sunday, April 25-26, 2009

	<b>TLC</b>	<b>343</b>	<b>347</b>
<b>Saturday morning</b>			
11:20 - 11:40	Doug Ray	Colton Magnant	Sosina Martirosyan Peterson
11:45 - 12:05	Eric Swartz	Valerie Hajdik	Mahmud Akelbek
<b>Saturday afternoon</b>			
2:40 - 3:00	Shanhzen Gao	Ji Li	Roberto Barrera
3:05 - 3:25	Shaun Sullivan	Vikram Kamat	Anthony Harrison
break			
3:50 - 4:10	Mahir Bilen Can	Art Duval	Svetlana Poznanovik
4:15 - 4:35	Daniela Ferrero	Ken W. Smith	Geir Helleloid
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5:00 - 5:20	Ernst L. Leiss	Jill Cochran	Kirsti Wash
<b>Sunday morning</b>			
10:00 - 10:20		Liangpan Li	Jay Bagga and Adrian Heinz

## Saturday morning, room TLC

11:20–11:40 **Doug Ray**, Texas State University  
*Cubic Cages*

Cages are graphs of specified regular degree and girth with the smallest possible order. A review of the current cubic cages will be given along with an algorithm for producing cages with a relatively small amount of information.

11:45–12:05 **Eric Swartz**, Ohio State University  
*2-Arc Transitive Polygonal Graphs of Large Girth and Valency*

A near-polygonal graph is a graph  $\Gamma$  which has a set  $\mathcal{C}$  of  $m$ -cycles for some positive integer  $m$  such that each 2-path of  $\Gamma$  is contained in exactly one cycle in  $\mathcal{C}$ . If  $m$  is the girth of  $\Gamma$  then the graph is called polygonal. Up until now, the only examples of 2-arc transitive polygonal graphs with arbitrarily large valency had girth no larger than seven, and the 2-arc transitive polygonal graph with largest girth had valency five and girth twenty-three (in fact, even with no restrictions on the automorphism group, there were no examples of polygonal graphs with odd girth greater than twenty-three). We provide a construction of an infinite family of polygonal graphs of arbitrary girth  $m$  with 2-arc transitive automorphism groups, showing that there are 2-arc transitive polygonal graphs of arbitrarily large valency for each girth  $m$ .

## Saturday morning, room 343

11:20–11:40 **Colton Magnant**, Lehigh University  
*Rainbow Ramsey Theory*

This talk will provide a brief survey of recent rainbow ramsey results and how they relate to the literature. Classical ramsey theory involves finding monochromatic structures in colored graphs. Similarly, rainbow ramsey theory involves finding either a monochromatic copy of a graph or a rainbow (totally multicolored) copy of another graph. For some rainbow graphs, for example the triangle, this question has been reasonably well studied. For most others, the problem is wide open.

11:45–12:05 **Valerie Hajdik**, University of Houston  
*Experiences with “undergraduate” version of Graffiti*

Graffiti is a computer program that makes conjectures in various sub-fields of mathematics and chemistry. Initial versions of Graffiti invented many conjectures that led to

research publications – many by well-known mathematicians – and subsequent version of this program were used by students to learn graph theory “Texas style”. I will discuss working on selected conjectures of Graffiti suitable for undergraduate students.

## Saturday morning, room 347

11:20–11:40 **Sosina Martirosyan Peterson**, University of Houston–Clear Lake

*Perfect Hash Families*

A perfect hash family,  $\text{PHF}(N; k, v, t)$  is an  $N \times k$  array with entries from a set of  $v$  symbols such that every  $N \times t$  sub-array contains at least one row having distinct symbols. Perfect hash families have been studied intensively in the last two decades as they find numerous applications in computer sciences and cryptography. They are also useful tools in constructions of covering arrays and other designs. Combinatorial methods are used to provide explicit constructions of perfect hash families with improved parameters. Several recursive constructions of perfect hash families are presented.

11:45–12:05 **Mahmud Akelbek**, Texas State University

*A bound on the scrambling index of primitive digraph in terms of boolean rank*

The scrambling index of a primitive digraph  $D$  is the smallest positive integer  $k$  such that for every pair of vertices  $u$  and  $v$ , there is a vertex  $w$  such that we can get to  $w$  from  $u$  and  $v$  in  $D$  by directed walks of length  $k$ ; it is denoted by  $k(D)$ .

For an  $m \times n$  boolean matrix  $M$ , its *boolean rank*  $b(M)$  is the smallest positive integer  $b$  such that for some  $m \times b$  boolean matrix  $A$  and  $b \times n$  boolean matrix  $B$ ,  $M = AB$ . The boolean rank of the zero matrix is defined to be zero.  $M = AB$  is called a *boolean rank factorization* of  $M$ .

In this talk, we present an upper bound on the scrambling index of primitive digraph  $D$  in terms of boolean rank  $b(A)$  of its adjacency matrix  $A(D)$ .

## Saturday afternoon, room TLC

2:40–3:00 **Shanzhen Gao**, Florida Atlantic University

*Some results and problems on self-avoiding walks*

A self-avoiding walk (SAW) is a sequence of moves on a lattice which does not visit the same point more than once. It was given as one of the two classical combinatorial problems in the Encyclopaedia Britannica. A SAW is interesting for simulations because

its properties cannot be calculated analytically. Calculating the number of self-avoiding walks in any given lattice is a common computational problem. We will present some interesting problems on SAWs and show you how to solve one problem.

3:05–3:25 **Shaun Sullivan**, Florida Atlantic University

*Counting Strings in Ballot Paths*

A ballot path stays weakly above the diagonal  $y = x$ , starts at the origin, and takes steps from the set  $\{\uparrow, \rightarrow\} = \{u, r\}$ . A pattern is a finite string made from the same step set; it is also a path. We consider  $b_{n,k}(m)$ , the number of ballot paths containing a given pattern  $k$  times reaching  $(n, m)$ . Certain types of patterns give sequences of polynomials that can be solved using multivariate Finite Operator Calculus. We only consider patterns  $p$  such that its reverse pattern  $\tilde{p}$  is a ballot path. We require this restriction so that the recurrence relation contains only values of the polynomial sequence that correspond to ballot paths and not the extensions of the polynomial sequence. For example, the pattern  $p = uuurr$  would give the recurrence  $b_{n,k}(m) = b_{n-1,k}(m) + b_{n,k}(m-1) - b_{n-2,k}(m-3) + b_{n-2,k-1}(m-3)$  when  $m > n$  and  $b_{n,k}(n) = b_{n-1,k}(n)$ , so if we used the first recurrence to define the polynomials, we would be using values below the diagonal that do not correspond to ballot paths. Notice that  $\tilde{p} = uurrr$  is not a ballot path. The patterns we consider here are called depth zero. To develop the recursions, we need to investigate the properties of the pattern we wish to avoid. Ballot paths reaching the diagonal can be viewed as Dyck paths, thus we are also counting strings in Dyck paths as a special case.

3:50–4:10 **Mahir Bilen Can**, Tulane University

*Symmetric functions and reductive monoids*

A  $J$ -irreducible reductive monoid is the (Zariski) closure of an irreducible representation of a reductive group  $G$ . In this talk, we will construct a family of symmetric functions for these monoids following Bergeron-Sottile construction of the (quasi)symmetric functions for a labeled graded poset.

4:15–4:35 **Daniela Ferrero**, Texas State University

*Balance and Connectivity in Signed Graphs*

The sign of a cycle in a sign graph is defined as the product of the signs of its edges. A signed graph is locally balanced at a given vertex if all the cycles that contain it, are positive. A signed graph is balanced if it is balanced at every vertex, or equivalently, if all of its cycles have positive sign. In this work we establish conditions for a signed graph to be balanced in terms of the connectivity of its underlying graph. Particularly,

we prove that if the underlying graph is highly connected, local balance implies balance of the signed graph. These results are applied to some families of signed graphs whose underlying graphs are line graphs.

5:00–5:20 **Ernst L. Leiss**, University of Houston

*Some Comments on the Towers of Hanoi Problem*

A number of years ago (a quarter of a century!), I took an interest in the Towers of Hanoi game. This is a problem that is frequently (ab)used in data structures and algorithms classes to illustrate the power of recursion. I subsequently generalized the game to be played on graphs; specifically, I assume a finite directed graph  $G = (V, E)$  with two distinguished nodes  $S$  and  $D$ , there are  $n$  disks of different sizes on node  $S$  such that no larger disk may lie on top of a smaller disk, and the objective is to move the  $n$  disks from  $S$  to  $D$  subject to the following rules:

1. Only one disk may be moved at a time and only along an edge in  $G$ .
2. A disk is always placed on top of all the disks on the node where it is moved and no larger disk may ever be placed on top of a smaller disk.

If the problem can be solved for a given graph for all  $n \geq 1$ , I call this Hanoi problem solvable. It turned out that there is a rather elegant characterization of all those graphs with solvable Hanoi problems (Leiss). If for a given graph the associated Hanoi problem is not solvable, I call it a finite Hanoi problem. It is an interesting question how many disks can be moved in finite Hanoi problems. It turns out that there are graphs where sub-exponentially many disks can be moved (Leiss; Azriel and Berend). A related question is how many moves are required. While the original Hanoi problem (the complete graph on three nodes) requires  $2n - 1$  moves, recent work (Azriel, Solomon and Solomon) indicates that significantly fewer moves may be required for some graphs with solvable Hanoi problems.

## Saturday afternoon, room 343

2:40–3:00 **Ji Li**, University of Arizona

*Fibonacci Compositions*

In Richard Stanley's Enumerative Combinatorics, Vol. 1, Exercise 14, page 46, we find several formulas expressing Fibonacci numbers in terms of sums over compositions. These formulas are easily proved using generating functions. Our goal is to study identities of this form systematically, and in particular, to answer the following questions: How can we find such identities? How can we prove them combinatorially?

In this talk, I will provide some (probably incomplete) answers to the above questions. The work is mainly done by Professor Ira Gessel. This talk will be accessible to anybody who maintains a basic knowledge of generating functions.

3:05–3:25 **Vikram Kamat**, Arizona State University

*Erdős-Ko-Rado theorems for chordal graphs and trees*

One of the more recent generalizations of the Erdős-Ko-Rado theorem, formulated by Holroyd, Spencer and Talbot, defines the Erdős-Ko-Rado property for graphs in the following manner: for a graph  $G$  and a positive integer  $r$ ,  $G$  is said to be  $r$ -EKR if no intersecting subfamily of the family of all independent vertex sets of size  $r$  is larger than the largest star, where a star centered at a vertex  $v$  is the family of all independent sets of size  $r$  containing  $v$ . In this talk, we present theorems which prove Erdős-Ko-Rado results for chordal graphs. We also consider the problem of finding maximum sized stars in trees. We conjecture that for any tree  $T$ , there is a maximum sized star centered at a leaf and prove this conjecture for  $r \leq 4$ . This is joint work with G. Hurlbert, Arizona State University.

3:50–4:10 **Art Duval**, University of Texas at El Paso

*Spanning trees and Laplacians of cubical complexes*

In previous work, we had extended the concept of a spanning tree from graphs to simplicial complexes; now we extend this idea further to CW-complexes, focusing especially on cubical complexes. Once again, if a complex satisfies a mild technical condition, then its spanning trees can be enumerated using its Laplacian matrices, generalizing the matrix-tree theorem. We apply this to enumerate the spanning trees of cubes and of their complete skeleta, by first computing their Laplacian eigenvalues, which turn out to be integral. We also define shifted cubical complexes, analogous to shifted simplicial complexes, and show that their Laplacian eigenvalues are integral as well.

This is joint work with Caroline Klivans and Jeremy Martin.

4:15–4:35 **Ken W. Smith**, Sam Houston State University

*Rational Idempotents in the Integral Group Ring and Their Applications*

Given a finite abelian group  $G$ , we construct a variety of combinatorial objects in  $G$  (such as difference sets and partial difference sets) using the rational idempotents of the group ring  $\mathbb{Q}[G]$ . Since combinatorial objects typically live in  $\mathbb{Z}[G]$ , these techniques provide divisibility conditions which dramatically narrow the search space for

the combinatorial object. In some cases, these techniques allow a full enumeration of the combinatorial objects or provide the means for an exhaustive search.

We give an introduction to this “idempotent” approach and provide a number of examples of objects constructed by this method.

5:00–5:20 **Jill Cochran**, Texas State University

*Finding and Visualizing Networks of Terrorism Buried in Large Data Sets*

The increasing availability of large amounts of information regarding terrorist activities and security of shipments amplifies the need for improved methods of analyzing and visualizing large data sets. We focus on the need to create accurate models to represent multidimensional data visually. These models are graphs, which represent networks of information derived from properties of the data or specifically related to geography or chronological sequences. Better visualization tools aid in preliminary data analysis. Then exploratory data analysis techniques such as clustering methods and graph theory techniques lead to better understandings of the data so that more informed decisions can be made in a timely manner. Tools have been developed to explore subnetworks or communities of interest by using clustering techniques related to minimum spanning trees and multidimensional scaling.

## Saturday afternoon, room 347

2:40–3:00 **Roberto Barrera**, Texas State University

*Power Domination in Cylinders and Tori*

A crucial task for electric power companies consists of the continuous monitoring of their power network. This monitoring can be efficiently accomplished by placing phase measurement units (PMUs) at selected network locations. However, due to the high cost of the PMUs, their number must be minimized. The power domination problem consists of finding the minimum number of PMUs needed to monitor a given power network, as well as to determine the locations where they should be placed. In terms of graphs, the problem consists of finding minimal sets of vertices that dominate the entire graph according to some given propagation rules imposed by the nature of the power network. The power dominating problem is NP-complete. However, closed formulas for the power domination number of certain families of graphs, such as rectangular grids have been found. We extend the results for grids to other families of graph products: the cylinders  $P_n \times C_m$  for integers  $n > 1$ ,  $m > 2$ , and the tori  $C_n \times C_m$  for integers  $n, m > 2$ . Joint work with D. Ferrero.

3:05–3:25 **Anthony Harrison**, Texas State University

*$L(2, 1)$ -labeling of hypercubes*

The channel assignment problem consists of assigning frequencies to transmitters so that the spectrum of frequencies used is minimized, while the communications do not interfere. Two transmitters are considered to interfere with each other if they share similar frequencies and are at a prescribed distance from one another.

The channel assignment problem translates into the following graph problem: find the minimum  $d$  such that the vertices of a given graph can be labeled with integers  $0, 1, \dots, d$  so that labels of vertices at a prescribed distance differ in a fixed amount. In particular, an  $L(2, 1)$ -labeling of a graph prescribes that labels at adjacent vertices must differ by at least 2 and that labels for vertices at distance 2 must differ by at least 1. This problem was found to be NP-complete. We find  $L(2, 1)$ -labelings for hypercubes that improve known upper bounds and show them to be minimal. Joint work with R. Barrera and D. Ferrero.

3:50–4:10 **Svetlana Poznanovik**, Texas A&M University

*Major Index for 01-Fillings of Moon Polyominoes*

In my talk I will present a definition for major index on 01-fillings of moon polyominoes. When specialized to certain shapes, this statistic reduces to the major index for permutations and set partitions. We consider the set  $\mathbf{F}(\mathcal{M}, \mathbf{s}; A)$  of all 01-fillings of a moon polyomino  $\mathcal{M}$  with given column sum  $\mathbf{s}$  whose empty rows are  $A$  and prove that this major index has the same distribution as the number of north-east chains, which are a natural extension of inversions (resp. crossings) for permutations (resp. set partitions). Hence our result generalizes the classical equidistribution result for the permutation statistics  $\text{inv}$  and  $\text{maj}$ . I will present two proofs: an algebraic one using generating functions and a bijective one in the spirit of Foata's second fundamental transformation on words and permutations (this is joint work with William Chen, Catherine Yan, and Arthur Yang).

4:15–4:35 **Geir Helleloid**, University of Texas at Austin

*Recognition and Reconstruction of Vertex-Edge Incidence Graphs*

In order to study the strong chromatic index of multigraphs, Brualdi and Massey introduced the incidence coloring number, which can be defined as the chromatic number of the vertex-edge incidence graph (VEIG) of a graph. VEIGs are reminiscent of line graphs, and much work has been done on structural characterizations and recognition algorithms for line graphs. I will discuss an analogous linear-time recognition algorithm for VEIGs that also permits the reconstruction of a graph from its VEIG. I will



also mention work on a forbidden subgraph characterization of the hereditary family of VEIGs of digraphs. This is joint work with Stephen Hartke.

5:00–5:20 **Kirsti Wash**, Texas State University

*On The Bounds of The Domination Number of Permutation Graphs*

For any permutation  $\alpha$  on the vertex set of a graph,  $G$ , the permutation graph  $P_\alpha(G)$  is obtained from two copies of  $G$ ,  $G$  and  $G'$ , by joining  $u \in V(G)$  and  $v \in V(G')$  if and only if  $\alpha(u) = v$ . Let  $\gamma(G)$  be the domination of  $G$ . It has been proven that for all permutations  $\alpha$  on any graph,

$$\gamma(G) \leq \gamma(P_\alpha(G)) \leq 2\gamma(G).$$

In this presentation, we focus on proving the following conjecture posited by Weizhen Gu (1999): For any permutation  $\alpha$  on the vertex set of a graph  $G$ ,  $\gamma(P_\alpha(G)) = \gamma(G)$  if and only if  $G$  consists only of isolated vertices.

## Sunday morning, room 343

10:00–10:20 **Liangpan Li**, Texas State University

*On the Chung-Erdős inequality and the Kochen-Stone inequality*

The Chung-Erdős inequality gives a lower bound on the amount of the union of finitely many finite sets if we know the amounts of all individual sets and all pairwise intersections. The Kochen-Stone inequality gives a lower bound on the probability of the upper limit of infinitely many events if we know the probabilities of all individual events and all pairwise intersections. In this talk, we generalize these two inequalities into weighted versions.

## Sunday morning, room 347

10:00–10:20 **Jay Bagga and Adrian Heinz**, Ball State University

*Graceful Labelings of Graphs*

Suppose that  $G$  is a graph with  $q$  edges. A one-one function  $f$  from the set of vertices  $V(G)$  to the set  $\{0, 1, 2, \dots, q\}$  induces an edge labeling where each edge  $uv$  is assigned the label  $|f(u) - f(v)|$ .  $f$  is called a *graceful labeling* if the induced edge labels are distinct. Thus the edge labels must be  $1, 2, \dots, q$ . In 1966, Rosa showed that a cycle  $C_n$  has a graceful labeling if and only if  $n \equiv 0 \pmod{4}$  or  $n \equiv 3 \pmod{4}$ . The famous *graceful tree conjecture* states that all trees have graceful labelings.

In this presentation, we demonstrate software systems for computing graceful labelings of trees and some classes of unicyclic graphs. We also investigate properties of such graceful labelings.