Networks: Shape and Function via Algebraic Topology

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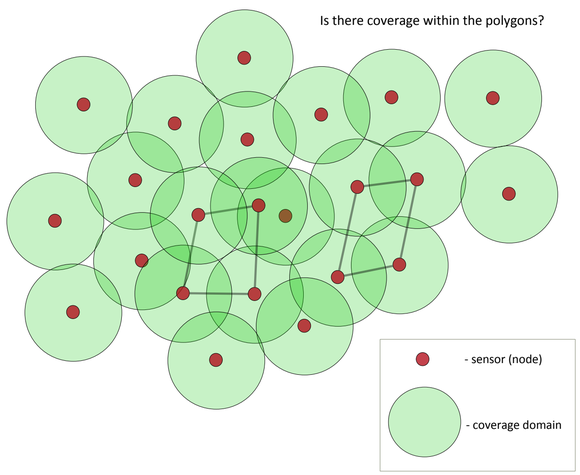
Introduction

Regardless of their function, individual sensors comprising a sensor network fall into two categories:

- complicated (and usually expensive) sensors equipped with the ability to collect global information
- sensors simplistic in design possessing limited or no ability to collect data about the structure of the network (i.e. no capacity for range-finding, location and/or orientation detection)

How does one answer questions about network coverage?

Is there coverage within the polygons?



Methodology

- If the location of each sensor is known precisely, then coverage questions can be answered effectively using computational geometry.
- Probabilistic methods can be employed if one assumes the sensors are distributed uniformly.

In reality, there are many practical applications in which these assumptions cannot be made.

- If the sensors are not enabled with GPS or similar abilities and not placed by hand, their locations will not be known.
- The assumption of uniform distribution is invalid if the sensors are dispersed on uneven terrain.

Suppose the sensors can communicate with each other over a short range.

Can local communication link data be used to determine global properties of the network?

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mathematical tools: algebraic topology and homology theory

Coverage

Setup

Given a collection of sensors with the following properties:

(1) the sensors have radially – symmetric sensing domains of radius r_s

(2) each sensor broadcasts a unique ID number with broadcasr range r_b

(3) every sensor receives the ID number from any sensor within broadcast range

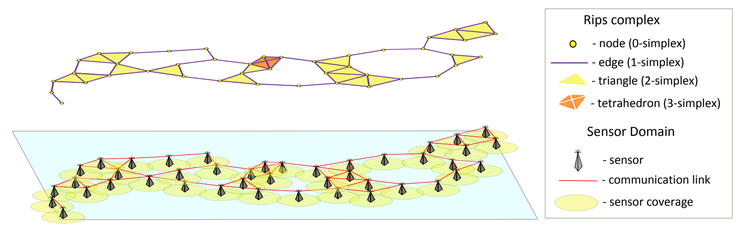
(4) a subcollection of the sensors forms the fence (boundary)

(5) sensor location/orientation is not known (we only have communication link data)

Question

Can the communication link data be used to determine coverage?

Yes! There exists a sufficient condition for coverage in terms of the homology of the Rips complex.



Redundant Coverage

In some cases, reducing or eliminating redundancy in network coverage is desirable.

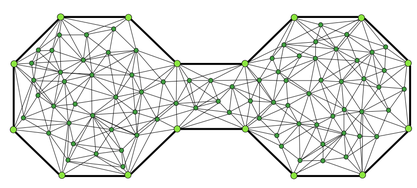
Examples

(1) power conservation

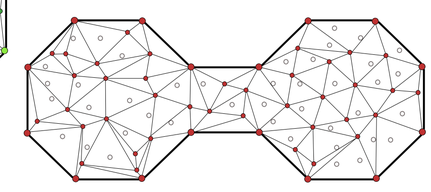
(2) computational efficiency

A special relative homology class of the Rips complex gives a minimal set of nodes for coverage.

This homology class can be computed algorithmically from the communication link data.



Rips complex projected onto the sensor network domain
- boundary node (0-simplex)
- interior node (0-simplex)
- edge (1-simplex)



Power-conserved network
- active sensor node
- sensor node in sleep mode
- relevant communication link

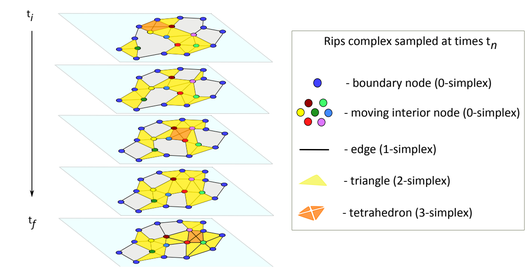
Wandering Coverage

Suppose that the sensors on the boundary are fixed but the interior sensors move.

Can wandering coverage be guaranteed?

Example – forest fire

- a sensor boundary would be set up on the ground surrounding an affected region
- sensors would be dropped from an aircraft into the region
- the sensors would be subject to movement by environmental factors both on deployment and after they have settled on initial locations



Assumptions

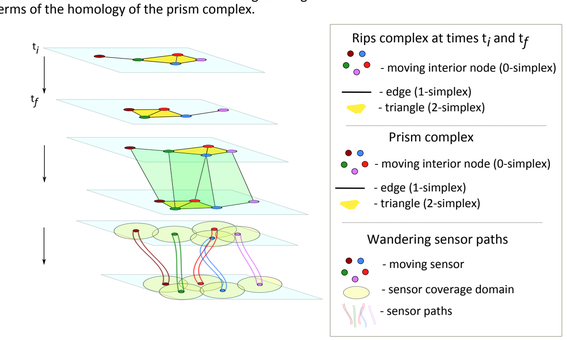
(1) sensors are as in the coverage problem

(2) fence sensors are fixed and always on-line

(3) interior sensors can move and go off-line temporarily

(4) the network communication link data is collected at a finite number of sampling times

There exists a sufficient condition for wandering coverage in terms of the homology of the prism complex.



Future Questions

- All of the mentioned problems assume that the communication link data is collected by a single global computational hub.
  What if the communication link data is collected by several local hubs?
  Can homology theory be used here?
  Would the sensors need to be equipped with additional capabilities? If so, what kind?
- Typically sensor ranges are not radially symmetric.
  How do we handle more general sensor ranges?
- Suppose a sensor network has random attributes (e.g. the sensor range).
  Can coverage questions be answered by developing a suitable random homology theory?