**Department of Mathematics**

**University of Houston**

# Analysis Seminar

## Friday, March 3, 2017

**13:00-14:00 – Room 646 PGH**

**Speaker:** N. Christopher Phillips (University of Oregon)

**Title:** An initial look at $L^p$ versions of von Neumann algebras

**Abstract:** A von Neumann algebra is a selfadjoint unital algebra of operators on a Hilbert space which is closed in the weak operator topology (equivalently, equal to its second commutant). Such algebras can be thought of as noncommutative analogs of the algebras $L^\infty(X, \mu)$ acting as multiplication operators on $L^2(X, \mu)$. They are much more special than C*-algebras, which generalize algebras of continuous functions on compact spaces.

We have looked in the past at various examples of "C*-like" algebras of operators on spaces of the form $L^p(X, \mu)$ for $p \in (1, \infty) \setminus \{2\}$. This talk is about an initial look at some examples of "von Neumann like" algebras on such spaces. It is joint work with Eusebio Gardella, and is very preliminary: the details of most proofs have not yet been written carefully and checked.

The most obvious example is all bounded operators on $L^p(X, \mu)$. After that, we will consider analogs on $L^p$ spaces of factors of type $\text{II}_1$. There are three common constructions of the (unique) hyperfinite factor of type $\text{II}_1$: as a direct limit of finite matrix algebras, as the group von Neumann algebra of a countable discrete group all of whose nontrivial conjugacy classes are infinite (under the additional assumption that the group is amenable), and as a crossed product by a free ergodic probability measure preserving action of a countable discrete group on a standard measure space (again, under the additional assumption that the group is amenable). All three constructions can be carried out on $L^p$. The first and third always give the same algebra, regardless of choices made. The second, however, gives a different algebra for every different choice of the group.

The algebras we get have at least some of the expected analogs of basic properties of the corresponding von Neumann algebras, while for other properties the analogs are often unknown, and are occasionally known to be false.
(This material is based upon work supported by the US National Science Foundation under Grants DMS-1101742 and DMS-1501144.)