Is there a Cuntz-Pimsner construction for $L^p$ operator algebras?

Abstract

For $p \in (1, \infty)$, and to some extent for $p = 1$, there is a recently initiated theory of operator algebras on $L^p$ spaces. Surprisingly, despite the lack of an adjoint, there are analogs with partially similar behavior of some of the standard examples in $C^*$-algebras, including AF algebras, Cuntz-Krieger algebras, full and reduced crossed products, groupoid $C^*$-algebras, and the Toeplitz algebra. There are also ways in which the behavior is quite different. For example, when $p \neq 2$ there is much more rigidity.

The Cuntz-Pimsner construction generalizes Cuntz-Krieger algebras and crossed products by $\mathbb{Z}$, both of which have $L^p$ operator algebra analogs. In one other case, an $L^p$ operator version has been done “by hand”, namely $L^2$ as a bimodule over the complex numbers. For $p \neq 2$, the algebras one gets from $l^p$ and $L^p([0,1])$ are not isomorphic to each other.

In this talk, I will give a brief introduction to $L^p$ operator algebras. Then I will describe some of the algebras corresponding to some cases of the Cuntz-Pimsner construction, describe ways in which they resemble and don’t resemble the corresponding $C^*$- algebras, say something about what has been done with these algebras, and state some open problems.