This presentation is accessible to graduate students and does not require any prior knowledge of the subject area.

Let $X$ be a set and $S, T : X \to X$. Two functions $S$ and $T \ast$-commute if $ST = TS$ and for every $(y, z) \in X \times X$ such that $S(y) = T(z)$, there exists a unique $x \in X$ such that $T(x) = y$ and $S(x) = z$. The concept of $\ast$-commuting maps was first introduced by Arzumanian and Renault where they studied $\ast$-commuting pairs of local homeomorphisms on a compact space $X$. Later Exel and Renault expand on this idea and provide many interesting examples. In this presentation I will discuss $\ast$-commuting maps in two settings: $k$-graphs and symbolic dynamics. This is joint work with Ben Maloney.

Higher-rank graphs (or $k$-graphs) were introduced by Kumjian and Pask to provide combinatorial models for the higher-rank Cuntz-Krieger $C^*$-algebras of Robertson and Steger. The shift maps on the infinite path space of a $k$-graph pairwise $\ast$-commute if and only if the $k$-graph is 1-coaligned in the sense that for each pair of paths $(e, f)$ with the same source there exists a pair of paths $(g, h)$ such that $ge = hf$. This equivalence is purely set-theoretic: the maps are not required to be continuous in any sense. When we restrict ourselves to a 2-graph formed from basic data, then the shifts $\ast$-commuting implies that the $C^*$-algebra of the 2-graph is simple and purely infinite.

Let $A$ be a finite alphabet and let $A^\mathbb{N}$ denote the one-sided infinite sequence space of elements in $A$. Morphisms between shift spaces are called sliding block codes, and any such morphism $\tau_d$ is built from a block map $d : A^n \to A$. A sliding block code is a local homeomorphism precisely when the sliding block code $\ast$-commutes with the shift map.