1. Define the doubling map $T(x) = 2x \mod 1$ on the unit interval with the Borel $\sigma$-algebra and Lebesgue measure.
   (a) Show that $T$ preserves Lebesgue measure.
   (b) Show that $T$ is isomorphic to the full one-sided shift on 2 symbols with the corresponding Borel $\sigma$-algebra and Bernoulli measure with $p_1 = p_2 = 1/2$.

2. Chávez’s transformation will be defined in terms of a cutting-and-stacking construction during lecture. Use the construction to argue the following:
   (a) The transformation is invertible.
   (b) The transformation preserves Lebesgue measure.
   (c) The transformation has continuous spectrum (which is equivalent to weak-mixing).
   (d) The transformation is not strong-mixing.

3. Let $n \in \mathbb{N}$ and let $X$ be the topological space with $n$ elements, taken with the discrete topology. Any permutation $\sigma \in S_n$ is then a continuous map from $X$ to itself. Describe, in terms of $\sigma$, the space $\mathcal{M}^\sigma(X)$ and the subset $\mathcal{E}^\sigma(X)$.

4. Let $X = [0, 1)$ and let $T : X \to X$ be the doubling map, defined above. Find a sequence $\{\mu_n\}_{n \in \mathbb{N}}$ of distinct elements of $\mathcal{E}^T(X)$ which converges in the weak-* topology to Lebesgue measure on $X$.

5. Let $T$ be a continuous transformation on a compact metric space $X$. Prove that any two distinct measures in $\mathcal{E}^T(X)$ are mutually singular.

6. Let $X$ be a compact metric space and $T : X \to X$ a continuous map. Prove that, for any $f \in C(X)$ and for any $x \in X$,

$$\inf_{\mu \in \mathcal{E}^T(X)} \int f \, d\mu \leq \liminf_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} f(T^n x).$$

7. (Harder) Give an example of a compact metric space $X$ and a continuous map $T : X \to X$, for which the set $\mathcal{E}^T(X)$ is not a closed subset of $\mathcal{M}^T(X)$.