MR0037469 (12,266g) 46.3X
Ionescu Tulcea, C. T.; **Marinescu, G.**
Théorie ergodique pour des classes d'opérations non complètement continues. (French)
Ann. of Math. (2) **52,** (1950). 140–147

Let B be a linear manifold in the linear vector space E over the complex numbers. Let B and E be Banach spaces under the norms $\|x\|, |x|$, respectively. It is assumed that $x_n \in B$, $\|x_n\| \leq K$, and $|x_n - x| \to 0$ imply that $x \in B$ and $\|x\| \leq K$. Let $C(B,E)$ be the subclass of all the bounded linear transformations in the space B determined by the following three conditions: (i) $|T^n x| \leq H|x|$, $x \in B$, $n = 1, 2, \cdots$, (ii) there exist positive constants R, r with $0 < r < 1$ such that $\|Tx\| \leq r\|x\| + R|x|$, $x \in B$, and (iii) TP is compact in E if P is bounded in B. Then for an operator T in this class $C(B,E)$ it is shown that there are at most a finite number of characteristic numbers $c_1, \cdots, c_p$ of absolute value 1 and that $T^n = \sum_{i=1}^p (1/c_i{}^n) T_i + S^n$, $n = 1, 2, \cdots$, where T_i is a projection in B with finite dimensional range, $T_i T_j = 0$, $T_i S = 0$, $i \neq j$, and $\|S^n\| \leq M\delta^n$, $n = 1, 2, \cdots$, with $\delta < 1$. [Cf. Doeblin and Fortet, Bull. Soc. Math. France **65,** 132–148 (1937); and the authors, C. R. Acad. Sci. Paris **227,** 667–669 (1948); MR0027468 (10,311e).]

N. Dunford

MR0027468 (10,311e) 60.0X
Ionescu, Tulcea; **Marinescu, G.**
Sur certaines chaînes à liaisons complètes. (French)
C. R. Acad. Sci. Paris **227,** (1948). 667–669

Let $P(t,A)$ be a function defined for t in a compact metric space and A a set of a Borel field of sets of a space E. For fixed t, $P(t,A)$ is completely additive in A, and $0 \leq P(t,A) \leq P(t,E) \leq 1$. For each $x \in E$, $y(t,x)$ is a function with range in t space. Both $P(t,A)$ and $y(t,x)$ have uniformly bounded difference quotients (bound less than 1) in t. Then the operator defined by $Tf = \int_E f[y(t,x)]P(t,dx)$ is a quasi-completely continuous operator on a certain Banach space of functions. The norms of the powers of T are uniformly bounded and 1 is a characteristic value if $P(t,E) = 1$. In the latter case the function $P(t,A)$ can be considered as a stochastic transition function, where t represents the past values assumed by a system, and A is the set into which the system is to go at the next transition. The known properties of the iterates of quasi-completely continuous operators then imply properties of the iterates of the given transition probabilities. This method was used, for t spaces with a finite number of points, by Doeblin and Fortet [Bull. Soc. Math. France **65,** 132–148 (1937)]. In particular, if the process is a Markov process, t space is E, and in this case the method was used, with less restrictive hypotheses, by Yosida and Kakutani [Ann. of Math. (2) **42,** 188–228 (1941); MR0003512 (2,230e)]. *J. L. Doob*