**MR591187 (82a:58038a)** 58F13 28D05 92A17

**Jakobson, M. V. [Yakobson, M. V.]**

**Construction of invariant measures absolutely continuous with respect to $dx$ for some maps of the interval.**

*Global theory of dynamical systems (Proc. Internat. Conf.,* … *Evanston, Ill., 1979), pp.* 246–257, *Lecture Notes in Math.*, 819, *Springer, Berlin*, 1980.



**MR586212 (82a:58038b)** 58F13 28D05 92A15

**Jakobson, M. V. [Yakobson, M. V.]**

**Invariant measures that are absolutely continuous with respect to $dx$ for one-parameter families of one-dimensional mappings. (Russian)**

*Uspekhi Mat. Nauk* **35** (1980), *no. 4(214)*, 215–216.

If $f\colon [0,1]\to[0,1]$ is a mapping and $\lambda$ a real number, denote $\lambda\cdot f$ by $f_\lambda$ and the set of all $\lambda$ for which there is an $f_\lambda$-invariant measure on $[0,1]$ which is absolutely continuous with respect to $dx$ by $M_f$. In the first paper for $f=x(x-1)$ proofs of the following facts are sketched: (1) The Lebesgue measure of $M_f$ is positive, and 4 is a Lebesgue point of $M_f$ in the sense that for sufficiently small $\delta>0$ the quotient of $\delta$ and the measure of $M_f\cap[4-\delta,4]$ approaches 1. (2) For each $\varepsilon>0$ there is a positive $K_0$ such that for $K\geq K_0$ the Lebesgue measure of $M_f\cap[K,K+4]$ is at least $4-\varepsilon$. In the second paper the following stronger results with the main idea of a proof are announced: (3) If $f$ is of class $C^3$, satisfies $f(0)=f(1)=0$, $f'(0)\neq 0$, and has only a finite number of critical points each of which is nondegenerate, then there is a number $T_0$ such that for each $\varepsilon>0$ there is a number $K_0$ with the following property: If $K\geq K_0$, then the measure of $M_f\cap[K,K+T_0]$ is at least $T_0-\varepsilon$. (4) For $f=x(x-1)$ the family $f_\lambda$ $(0\leq\lambda\leq 4)$ has a $C^3$ neighborhood $U$ in the space of all $C^3$ families $[0,4]\to C^3([0,1],[0,1])$ such that for each $g_\lambda\in U$ the set $\{\lambda;\ [0,1]$ has a $g_\lambda$-invariant measure which is absolutely continuous with respect to $dx\}$ has positive Lebesgue measure. These results confirm a conjecture of Ruelle and Sinaĭ.

{For the entire collection in which the first paper appears see MR0591170 (81j:58002).}

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