Abstract. The main goal of this lecture is to discuss the numerical solution of the following ***inverse problem*** (from Geophysics) for the ***Eikonal equation***:

$$\text{(IP-E)} \quad \begin{cases} \text{Find } c_{\text{opt}} \in \mathscr{C}, \text{ such that} \\ J(c_{\text{opt}}) \leq J(c), \forall c \in \mathscr{C}, \end{cases}$$

where

$$\mathscr{C} = \{c \mid c \in L^\infty(\Omega), 0 < c_{\min} \leq c \leq c_{\max}\},$$

$$J(c) = \frac{1}{2}\int_\Gamma | z - y_{\text{meas}} |^2 d\Gamma,$$

z (the time of first arrival) being the solution of the following Eikonal problem

$$\begin{cases} |\nabla z| = \dfrac{1}{c} \text{ in } \Omega, \\ z \geq 0, \\ z(x_s) = 0. \end{cases}$$

Above, $\Omega \subset \mathbf{R}^d$ $(d \geq 2)$, $\Gamma \subset \overline{\Omega}$, $x_s \in \overline{\Omega}$ being the point source of the wave, In practice, several sources are used to reconstruct c_{opt}.

In order to solve the above inverse problem, we advocate a methodology combining external penalty, harmonic and bi-harmonic regularizations, operator-splitting to decouple nonlinearities and differential operators, and finite element methods. The results of some numerical experiments will be presented.