

## PROBLEM OF THE WEEK - FALL 2014 - WEEK 1

### QUESTION 1

We know that

$$1^2 = 1$$

$$1^2 + 2^2 = 1 + 4 = 5$$

$$1^2 + 2^2 + 3^2 = 5 + 9 = 14.$$

But what is

$$\sum_{k=1}^{50} k^2 = 1^2 + 2^2 + 3^2 + 4^2 + \dots + 47^2 + 48^2 + 49^2 + 50^2 ?$$

### QUESTION 2

Consider the function  $f : (0, \infty) \rightarrow \mathbb{R}$  defined by  $f(x) = x \coth(x)$ . Show that  $f$  is strictly increasing. Note:

$$\coth(x) = \frac{\cosh(x)}{\sinh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}}.$$