

PROBLEM OF THE WEEK - FALL 2014 - WEEK 5

QUESTION 1

Tic-Tac-Toe is commonly played on a 3×3 grid, and it is known that if both players make optimal moves then it will result in a tie; i.e., one player cannot force a win.

- (i) Can either player force a win in 4×4 tic-tac-toe (where a win is four in a row, four in a column, or four along one of the two main diagonals)?
- (ii) (BONUS) Can either player force a win in $n \times n$ tic-tac-toe, where $n > 4$?

QUESTION 2

Consider the following matrix over the ring of integers modulo 10:

$$A = \begin{bmatrix} 1 & 1 & 1 & 2 & 1 \\ 9 & 1 & 9 & 0 & 9 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 2 & 1 & 3 & 1 \\ 0 & 2 & 0 & 2 & 0 \end{bmatrix}$$

- (i) What is the characteristic polynomial for A ?
- (ii) What is the null ideal of A , i.e., the set of polynomials that annihilate A ?