## PROBLEM OF THE WEEK - FALL 2014 - WEEK 7

## QUESTION 1

[Fitting a polynomial to data.]

(a) Say we have n points  $\{(a_1, b_1), ..., (a_n, b_n)\}$  with the y-coordinates all distinct and the xcoordinates all distinct. Give an explicit formula for the polynomial whose graph goes through all of the points.

For example, given  $\{(1,0), (0,-1)\}$  we have  $p(x) = x^2 - 1$ , since p(1) = 0 and p(0) = -1.

(b) (BONUS) Can we do the same if we are given infinitely many points? If not, give a counterexample.

## QUESTION 2

[Fibonacci Fun] Give a proof of the closed formula for the Fibonacci numbers: Let  $F_0 = F_1 = 0$ and  $F_n = F_{n-2} + F_{n-1}$  for  $n \ge 2$ . Then

$$F_N = \frac{1}{\sqrt{5}} \left[ \left( \frac{1+\sqrt{5}}{2} \right)^{n+1} - \left( \frac{1-\sqrt{5}}{2} \right)^{n+1} \right]$$