PROBLEM OF THE WEEK - FALL 2014 - WEEK OF 11-19-14

QUESTION 1

Consider the expression:
$$\frac{1}{\pi^2} \int_0^\infty \frac{\log(x)}{1+x^n} dx.$$
For which integers $n \geq 2$ is this expression a rational number?

QUESTION 2

Let $\gamma : [0,1] \to \mathbb{R}^2$ be a closed curve. Show that there must exist four points on the curve $P, Q, R, S \in \gamma([0,1])$ that form the vertices of a rectangle.

For example, if $\gamma(t) = (2\sin 2\pi t, \cos 2\pi t)$ the points given by $t = \frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{11}{12}$ form a rectangle: