Problem 5.1: To begin, we will prove a lemma. Consider an $m \times m$ board (with $m$ possibly infinite), and a game where two players, I and II, take turns marking squares, and the first player marks $N$ squares in a row, column or diagonal wins.

Lemma 1: The second player, II, does not have a winning strategy.

Proof: Suppose player II does have a winning strategy. After player I makes the first move by marking an arbitrary spot, player I becomes the second player, and can simply execute the winning strategy, and thus win before player II. So long, player I does not run into the case where he/she needs to mark a now-occupied spot. However, this does not affect player I’s strategy, as he/she can mark another arbitrary square. The extra square cannot adversely impact player I in the execution of the winning strategy.

Now we will prove the claim.

Claim 2: A game of a $4 \times 4$ tic-tac-toe will always end in a draw if both players play optimally.

Proof: We know from Lemma 1 that player II does not have a winning strategy. What we need to show is that player I does not have one either as player II does have a drawing strategy. Up to symmetry, player I has three possible first moves: (corner, center-square, outer-edge)

Regardless of the position of the first $\times$, player II can always place his $\bigcirc$ in the inner 4 squares. Note that there are 10 possible winning possibilities (4 horizontal, 4 vertical, and 2 diagonals). However, once player II places the $\bigcirc$, three of the ten options can no longer allow player I to win, with 14 squares left.

At this point, we can just come up with a matching move strategy. For every move player I makes, maker II can make a blocking move that close down at least two (possible three) winning options for player I. Hence, player II can always force a draw by putting a $\bigcirc$ occupying every possible winning line.

To be more precise, consider the the scenarios above:

If Player I puts an $\times$ into the square labeled by the letter Z, Player II can always put a matching $\bigcirc$ into the other square label with the same letter Z, causing a draw.