

**Problem 8.1:** Can you tell exactly what  $k$  is? If not, what's the smallest number  $k$  can be?

**Solution.**  $k = 420n + 119$  for integer  $k \geq 0$ .

For  $n \in \{2, 3, 4, 5, 6\}$ ,  $k \equiv n - 1 \pmod{n}$ , or  $k + 1 \equiv n \equiv 0 \pmod{n}$ . We know 2, 3, 4, 5, 6 divide  $k + 1$ , hence,  $k + 1$  is a multiple of  $\text{lcm}(2, 3, 4, 5, 6) = 60$ . However, we know  $7 \mid k = 60m - 1$ . This means  $4m - 1 \equiv 0 \pmod{7}$ . The only  $m$  where this equation holds is if  $m \equiv 2 \pmod{7}$ , or  $m$  of form  $7n + 2$ . Therefore, any number of the form  $60(7n + 2) - 1 = 420n + 119$ ,  $n \in \mathbb{N} \cup \{0\}$  would work.

**Remark:** The smallest  $k$  is when  $n = 0$ , or  $k = 119$ .