## **Texas PDE Conference - University of Houston, 2015**

## **Book of Abstracts**

1. ------

Speaker: Reza Ahangar - Texas A & M University, Kingsville

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*Title:* Optimal Control Solution to Nonlinear Causal Operator Systems with Target State

**Abstract:** We will study the optimal control solution satisfying nonlinear causal operator differential equations. The transformation by these operators depends on the information about the trajectory as a function of past history which will not be affected by future events. These dynamical systems involve a large class of delay, integral, composition, or Cartesian products of operators. The optimal solutions for operator dynamical systems with target states will be studied.

2. \_\_\_\_\_

Speaker: Dambaru Bhatta - Department of Mathematics, University of Texas-Pan American

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Coauthor: Daniel N. Riahi - Department of Mathematics, University of Texas-Pan American

*Title:* Computation of the vertical velocity components for a three dimensional convective flow in a mushy layer

**Abstract:** Here we consider a nonlinear three dimensional convective flow in a mushy layer which is cooled from below. During solidification, fluid flow within the mushy layer can cause vertical chimneys or channels void of solid. These chimneys can generate imperfections in the final form of the solidified alloy. A quadratic nonlinear evolution equation satisfied by the amplitude is derived for the hexagonal cells. Then we compute the linear and first order vertical velocity components for super and sub critical cases.

2	
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Speaker: Youn-Sha Chan - Department of Mathematics and Statistics, University of Houston-Downtown

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Title: Crack Problem Used as a Criterion for Examining Elasticity Theories

**Abstract:** Various elasticity theories including linear and nonlinear theories on material modeling are reviewed and examined in light of a crack problem.

The crack problem is formulated as a boundary value problem, and a structure of hierarchy in the governing partial differential equations (PDEs) are observed. The structure of PDE hierarchy includes two sets of comparisons:

(1) homogeneous materials versus nonhomogeneous materials, and

(2) classical linear elasticity theory versus strain-gradient elasticity theory.

We then found that crack problems can be used to simplify the challenging higher-order of the governing PDEs. In addition, crack problems under each elasticity theory are formulated and their solutions are also presented. We also show that the fourth order PDE in the higher order strain-gradient elasticity theory converges to the second order PDE in classical linear elastic fracture mechanics (CLEFM). In the case of nonlinear theories, we observe that some nonlinear elasticity theory may not be applicable to formulate crack problems.

4. -----

Speaker: Benito Chen-Charpentier - Department of Mathematics, University of Texas at Arlington

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Title: Biofilm Growth on Medical Implants with Randomness

**Abstract:** Biofilms are colonies of bacteria that attach to surfaces by producing extracellular polymer substances. They may cause serious infections in humans and animals, and also cause problems in hydraulic machinery. On the other hand, they can also be used to control pollution in underground aquifers by forming biobarriers.

In this paper we model the growth of a biofilm established on a medical implant. We assume that the biofilm growth is given by a logistic reaction term with the growth rate being a random variable with a given distribution. This way we take into account the variability in the bacterial populations, and the measurement and experimental errors. The diffusion coefficient of the microbes is also taken to be random. A stochastic spectral representation of the parameters and the unknown stochastic process is used, together with the polynomial chaos method, to obtain a system of partial differential equations, which is integrated numerically to obtain the evolution of the mean and higher-order moments with respect to time. Some examples are presented.

**Speaker:** Goong Chen - Department of Mathematics and Institute for Quantum Science and Engineering Texas A&M University, College Station

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*Title:* Malaysia Airlines Flight MH370: A Modeling and Numerical Simulation Study of Airplane Crash and Water Landing

**Abstract:** On March 8, 2014, Malaysia Airlines flight MH370 disappeared less than an hour after take-off on a route from Kuala Lumpur to Beijing. Its mysterious fate is one of the most intriguing stories of the year 2014, but the available evidence has indicated that the airliner has crashed into the Indian Ocean. Its search and recovery operation also has constituted the most expensive one in the aviation history so far.

In this talk, the speaker will first revisit the study of crashing and ditching of aircraft into the ocean as a classical water-entry problem in applied mathematics. Then the entry of an airliner into the ocean will be modeled as a two-phase fluid-structure integration problem with compressible aero-hydrodynamics and six-degree of freedom of motion. Numerical simulations are performed by using the OpenFOAM software. Several video simulations of dynamic motion of an airliner flying into the ocean will be shown. Impact damage will also be assessed based on the analysis of the Space Shuttle Challenger disaster. This is joint work by G. Chen, C. Gu, P.J. Morris, E.G. Paterson, A. Sergeev, Y.-C. Wang and T. Wierzbicki.

6. -----

Speaker: Manki Cho - Department of mathematics, University of Houston

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Title: Steklov eigenproblems and approximations of harmonic functions

**Abstract:** We provide the value of a harmonic function at the center of a rectangle that involve boundary integrals. The central value of a harmonic function is shown to be well approximated by the mean value of the function on the boundary plus a very small number (often just 1 or 2) of additional boundary integrals. These are consequences of Steklov (spectral) representations of the functions that converge exponentially at the center. Similar approximation are found for the central values of solutions of Robin and Neumann boundary value problems. The results are based on explicit expressions for the Steklov eigenvalues and eigenfunctions. This is joint work with Professor Giles Auchmuty.

Speaker: Lokenath Debnath - Department of Mathematics, University of Texas-Pan American

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Title: Cylindrical KdV Equations and Applications to Ocean Waves

*Abstract:* This paper deals with the cylindrical KdV equations and their solutions by series and multiple scales methods.

8. -----

Speaker: Eleftherios Gkioulekas - Department of Mathematics University of Texas - Pan American

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*Title:* Revisiting the dissipation scales of the energy cascade of 3D turbulence as anomalous scaling functions

**Abstract:** The usual concept of an energy cascade that has a unique associated dissipation scale is an oversimplification. Aside from the fact that self-similar scaling for higher-order structure functions terminates at different dissipation scales, back in 1996, L'vov and Procaccia noted an additional anomaly; starting from an  $n^{th}$ - order generalized structure function, consisting of a product of velocity differences, each between two different points, when all velocity difference separations have length scale *R* and one velocity difference separation is reduced to a smaller scale *r*, the crossover to dissipation range will occur at the scale \$-lell\_n (R)\$ which is *R*-dependent. The fixed point  $\lambda_n$  such that \$-lell\_n (\lambda\_n)=\lambda\_n\$ gives the standard dissipation scale associated with the  $n^{th}$ - order standard structure functions. In my talk, I will make note of an additional anomaly. If, instead of reducing one velocity difference separation, we reduce *p* velocity difference separations to scale *r*, that defines a different dissipation scale function \$ and a different fixed-point  $\lambda_{np}$ . The new anomaly is that  $\lambda_{np}$  is not independent of *p*.

9. -----

Speaker: Yuliya Gorb - Department of Mathematics, University of Houston

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Title: Approximation of Dirichlet-to-Neumann Map in a High Contrast Conductivity Problem

**Abstract:** A model of a composite material consisting of a matrix of finite conductivity with ideally conducting almost touching particles is considered, and a discrete network approximation for the Dirichlet-to-Neumann map is constructed and justified.

10. -----

Speaker: Sergey Grigorian - Department of Mathematics University of Texas - Pan American

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Title: Soliton solutions of the modified Laplacian coflow of G2-structures on manifolds with symmetry

**Abstract:** G2-structures on 7-dimensional manifolds play a very important role in both geometry and physics. One of the ways of better understanding the relationships between different types of G2-structures is to study their flows. A natural flow of G2-structures is an analog of the heat equation. However since the Laplacian is determined by the G2-structure itself, this becomes a nonlinear PDE. Here we analyze such a flow when the 7-manifold is a warped product of a 6-dimensional Calabi-Yau or nearly Kähler manifold and either a circle or an interval, in which case the equations reduce to a particular system of nonlinear PDEs with three dependent and two independent variables. We then look at the soliton equations for this system, where in some cases explicit non-trivial solutions are obtained.

11. -----

Speaker: Haicheng Gu - Department of Mathematics, University of Texas - Pan American, TX

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Title: An Analysis of Financial Data with Wavelets and Fractal Functions

**Abstract:** The study of fractal functions starts from Weierstrass's nowhere differentiable function and beyond. Seemingly different types of nowhere differentiable functions are unified under the fractal point of view. This unification led to new mathematical methods and applications in areas that includes: dimension theory, dynamical systems and chaotic dynamics, image analysis, and wavelet theory. Now Wavelet analysis is an exciting new method for solving difficult problems in mathematics, physics, and engineering, with modern applications as diverse as wave propagation, data compression, image processing and financial engineering.

In this paper, our goal is to find the deeper connection between fractal functions and wavelets, and find the application in finance to make predictions as to the risk involved for particular stocks or options.

Speaker: Cong Gu - Texas A&M University, College Station

Title: Global Plastic Bending of Aircraft Fuselage

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**Abstract:** In the problem of aircraft water entry, the impact is often found not large enough for an immediate local destruction like in the case of crashing on land. However, it can still induce an excess of bending somewhere along

the fuselage, which may cause severe damage and even disintegration of the aircraft into multiple parts. In this talk, the speaker will model the aircraft fuselage as a plastic free-free beam. Beam theories and equations involved will be discussed. The model will then be applied to the data obtained from CFD simulations of aircraft water entry to study the dynamic structural failure of the aircraft in this process.

13. -----

Speaker: Natali Hritonenko - Department of Mathematics, Prairie View A&M University

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Coauthor: Yuri Yatsenko - College of Business and Economics, Houston Baptist University

Title: Optimal Control of Differential Equations Arising in Environmental Protection

**Abstract:** Sustainable development and environmental protection are among important current issues. Different modeling approaches have been suggested to address related management problems. After their brief survey, a dynamic economic-environmental model is discussed in more details. It aims to investigate a role of mitigation and adaptation to reduce negative climate changes and rational investment into various environmental protection means which is essential for designing long-term environmental policies and regulations on international, national, and local levels. A qualitative analysis of the model demonstrates the existence of a unique steady state and leads to determining the optimal balance between investing into environmental adaptation and emission abatement depending on a country development level and its contribution to a global pollution. The model is calibrated on the available economic and climate data and policies. Further research directions and open questions are highlighted.

Speaker: Qiaoyi Hu - Department of Mathematics, South China Agricultural University,

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Ji Lin - College of Engineering, South China Agricultural University, Guangzhou, China

Title: Initial boundary value problem for a coupled Camassa-Holm system with peakons

**Abstract:** We investigate the homogeneous initial boundary value problem for a coupled Camassa-Holm system with peakons on the half line. We first establish the local well-posedness for the system. We then present a precise blowup scenario and several blowup results of strong solutions to the system. We finally give the blowup rate of strong solutions to the system when blowup occurs.

15. -----

Speaker: Mark Hubenthal - Department of Mathematics, University of Houston

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Coauthor: Daniel Onofrei, Department of Mathematics, University of Houston

*Title:* Sensitivity analysis for active control of the Helmholtz equation.

**Abstract:** We consider the problem of active exterior cloaking for the Helmholtz equation. The previous work of Onofrei implies the following: Suppose we have a source region  $D_a$  subset of  $R^d$  (d equals 2 or 3) and a solution  $u_0$  to the homogeneous scalar Helmholtz equation in a set containing the control region  $D_c$  subset of  $R^d$ . Then there exists an infinite class of boundary data on  $\partial D_a$  such that the radiating solution to the corresponding exterior scalar Helmholtz problem in  $R^d \setminus D_a$  will closely approximate  $u_0$  in  $D_c$  while having vanishingly small values beyond a sufficiently large ``far-field" radius R. In this work we study the minimal energy solution to the above problem, which is obtained using Tikhonov regularization and the Morozov discrepancy principle, and we perform a detailed sensitivity analysis. That is, we analyze the stability of the minimal energy solution with respect to measurement errors as well as the feasibility of the active scheme (power budget, accuracy, stability) depending on: mutual distances between the antenna, control region and far field radius R, regularization parameter, frequency, location of the source, etc. This is joint work with Daniel Onofrei.

Speaker: Akif Ibragimov - Department of Mathematics and Statistics, Texas Tech University

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*Title:* Qualitative Properties of the solution of elliptic equation of second order in non-divergent form with respect to Zaremba type boundary problem.

**Abstract:** In the talk I will present results obtained recently with professor Nazarov from St. Petersburg Department of Steklov Mathematical Institute of Russian Academy of Sciences. We will study behavior of the solution of elliptic equation in non-divergent form near boundary point, which is junction of the Dirichlet and Neumann (with oblique derivatives) boundary Data. First result for regularity of the point of junction of Dirchlet and Neumann boundary was obtained for equation in divergent form by professor Mazya. His technique is not applicable for equation in non-divergent form. In our case Neumann boundary assumed to be only Lipchitz or specific "funnel" type structure. Under this summation problem cannot be reduced to Dirichlet problem and require particular barrier technique. We use concept of Landis type Lemma of Growth to prove Wiener type test for regularity of the point of junction of Dirichlet and Neumann boundaries. Regularity condition is formulated in term of Wiener type series with s-capacity depending on Dirichlet Boundary only.

17. -----

Speaker: Katarina Jegdic - Department of Mathematics and Statistics, University of Houston – Downtown

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Title: Semi-hyperbolic patches for the unsteady transonic small disturbance equation

**Abstract:** We consider a two-dimensional Riemann problem for the unsteady transonic small disturbance equation resulting in diverging rarefaction waves. We write the problem in self-similar coordinates and we obtain a mixed type (hyperbolic-elliptic) system. Resolving the one-dimensional discontinuities in the far field, where the system is hyperbolic, and using characteristics, we formulate the problem in a semi-hyperbolic patch that is between the hyperbolic and the elliptic regions. A semi-hyperbolic patch is known as a region where one family out of two nonlinear families of characteristics starts on a sonic curve and ends on a transonic shock. We obtain existence of a smooth local solution in this semi-hyperbolic patch and we prove various properties of global smooth solutions based on a characteristic decomposition using directional derivatives. This is joint work with I. Jegdic.

**Speaker:** Steven D. London - Department of Mathematics and Statistics, University of Houston-Downtown

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*Title:* Weakly Nonlinear Hydrodynamic Waves, Mean Flows and Instabilities in a Keplerian Accretion Disk

**Abstract:** The universe is full of galaxies, stars and planets: gravitationally bound, compact structures that started out as well dispersed bodies of gas and dust (accretion disks). However, even a small amount of angular momentum, which is conserved, would strongly restrict any contraction of this material. It has been shown that, in the presence of a magnetic field, instabilities can develop in accretion disks which should allow this angular momentum constraint to be overcome, allowing the disk material to contract. On the other hand, the situation in the hydrodynamic case (no magnetic field) is somewhat unclear. The purpose of the work described here is indicate a possible mechanism for accretion disk contraction in the hydrodynamic case for Keplerian accretion disks (in this case, disk material in uniform circular motion, subject to Newton's inverse square law of gravity).

The equations for a hydrodynamic (non- electrically conducting) fluid in cylindrical coordinates are used to study weakly nonlinear waves in a Keplerian accretion disk. The method of geometric optics (ray method) is applied in order to study these waves in a global setting. The equations are expanded about an ambient state in which the fluid particles are in equilibrium with a gravitational force which is a function of the cylindrical radius only; the equilibrium angular velocity is constant on cylinders and self-gravitation is ignored. The perturbation terms are small but the nonlinear terms in the equations are retained. Approximations to the governing equations are found in the form of wave trains which are slowly varying in time and space. At leading order, a dispersion relation is found in which the wave propagation occurs either inside of outside a bounding circle. At first order, ordinary differential equations are obtained for the wave amplitude functions. With the right initial conditions, these amplitude functions can be shown to be unstable on a relatively long time scale. Equations are found for the mean flow quantities. In particular, the radial mean flow can be shown to flow inward toward the massive central object for waves propagating in a particular direction. These flows may also grow on relatively long time scales. This work appears to illustrate a mechanism for overcoming the angular momentum constraint on the collapse of accretion disk material in which no magnetic field is present.

19. -----

Speaker: Thanuka W. Pathiranage - Department of Mathematics and Statistics, Texas Tech University

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*Title:* Analysis of the Error in an Iterative Algorithm for Solution of the Regulator Equations for Linear Distributed Parameter Control Systems

**Abstract:** The regulator equations are a coupled pair of operator equations that arise in the geometric approach to regulation in systems and control. The main problems of control for this work consist of asymptotic tracking and disturbance rejection for linear parabolic distributed parameter systems. Our approach to solving problems of this type is geometric regulation in which control laws are obtained by solving the regulator equations. In general it is not easy to solve the regulator equations or even obtain accurate numerical solutions. In this paper we present the  $\beta$ -iteration method for obtaining approximate solutions of regulator equations for infinite dimensional linear control systems with input and output operators in the Hilbert state space. A major advantage of this theory compared to previous works is that it can be applied to any smooth reference signal and an explicit error analysis is available for each step in the iteration. In this work, we describe the  $\beta$ -iteration method and present an analysis of the error for the iterative method. We also give theoretical estimates for its convergence. The convergence of the iterative method depends on the parameter  $\beta$ , ( $0 < \beta < 1$ ) and also on the exponential stability of the C<sub>0</sub> semigroup generated by the open loop plant.

20. -----

Speaker: Sara Pollock - Texas A&M University, College Station

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Title: Adaptive regularization strategies for nonlinear PDE.

**Abstract:** We will discuss an adaptive regularization strategy for the numerical approximation of stationary quasilinear diffusion and convection diffusion problems. We will relate the PDE on a coarse mesh to a noisy ill-posed problem and discuss the use of adaptive regularization for stabilizing Newton-like iterations with an ill-conditioned and possibly indefinite Jacobian. The stabilized method will be set in the context of an adaptive finite element method, where transitional configurations of the regularized coarse-mesh iterates are used with standard error indicators to determine both adaptive mesh partitioning and selective regularization. Numerical examples demonstrate the effectiveness of the method and illustrate the distinct phases of the solution process.

21. -----

Speaker: Zhijun Qiao - Department of Mathematics University of Texas - Pan American, Edinburg

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Title: A synthetical two-component model with peakon solutions

**Abstract:** A generalized two-component model with peakon solutions is proposed in this talk. It allows an arbitrary function to be involved in as well as including some existing integrable peakon equations as special reductions. The generalized two-component system is shown to possess Lax pair and infinitely many conservation laws. Bi-Hamiltonian structures and peakon interactions are discussed in detail for typical representative equations of the generalized system. In particular, a new type of N-peakon solution, which is not in the traveling wave type, is obtained from the generalized system.

22. ------

Speaker: Mauricio A. Rivas - Department of Mathematics, Wake Forest University, NC

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*Title:* Linear Elliptic Eigenproblems and Observing Lyapunov Exponents of Infinite-Dimensional Dynamic

**Abstract:** We address the question: to what extent do inferences about differentiable infinitedimensional dynamical systems, which may arise from evolution PDEs, follow from projecting the dynamics into  $\mathbf{R}^{N}$  using 'typical' nonlinear maps? This is joint work with Professor William Ott. Related to this question are sets of determining modes. We discuss a method for obtaining eigenfunctions of some linear elliptic eigenvalue problems that may include Robin or Steklov boundary conditions. This is joint work with Professor Giles Auchmuty.

23. -----

Speaker: Ranadhir Roy - Mathematics Department, University of Texas Reo Grande Valley, TX

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Title: Sensitivity Analysis of Drug Distribution in a Brain Tumor using Simulated Mathematical Model

**Abstract:** Sensitivity analysis is an effective tool for systematically identifying specific perturbations in parameters that have significant effects on the behavior of a given bio-system. In this work, a computational model based on diffusion and convection processes is developed that allows sensitivity analysis of anti-cancer drug distribution in a brain tumor due to perturbations of parameters like vessel permeability and diffusion coefficient. Each of these sensitive parameters exhibited a similarly changing pattern in that a relatively larger increase or decrease in their value resulted in a corresponding anticancer drug effect. This model incorporates heterogeneous vasculature and tissue porosity to account for non-uniform distribution of drug concentration. Overall, the algorithm operated reliably over relatively large variations of parameters, hence confirming the robustness of the model. The sensitivity analysis shows that permeability has more impact than the diffusion coefficient, and it also demonstrates that among diffusion and convection mechanism of drug transport, diffusion is dominant.

24	

Speaker: Pablo Raúl Stinga - Department of Mathematics, University of Texas at Austin

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Title: Regularity theory for fractional equations

**Abstract:** The fractional Laplacian is a classical object in Harmonic and Functional Analysis. Nevertheless, some fine tools needed in the study of PDEs like Harnack inequalities and Schauder regularity estimates are not available from such a general theory. L. Caffarelli and L. Silvestre introduced the nowadays famous extension problem for the fractional Laplacian. This turned out to be a powerful technique for the theory of fractional equations.

We will explain a novel point of view to handle fractional operators: the semigroup language approach. The method was introduced in my PhD thesis (2010). With this we can understand what a fractional operator is and we can generalize the Caffarelli-Silvestre extension problem to a large class of fractional operators. Applications to Hölder regularity estimates for fractional divergence form elliptic operators will be shown.

25. -----

Speaker: Eric Tovar - Department of Mathematics, University of Texas-Pan American

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Coauthor: Zhijun Qiao - Department of Mathematics, University of Texas-Pan American

Title: Peakon Solutions for (2 + 1)-dimensional General Camassa-Holm Equation

**Abstract:** In recent years, the Camassa-Holm equation has attracted much attention to the theory of integrable systems and solitons. In this study, we generalize the CH equation to (2 + 1)- dimensions and derive single peakon and multi-peakon solutions. We also look at other topics such as smooth soliton solutions, cuspons and peakon stability.

26. -----

Speaker: Ray Treinen - Department of Mathematics, Texas State University, San Marcos

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*Title:* On the classification and asymptotic behavior of the symmetric capillary surfaces.

**Abstract:** The symmetric configurations for the equilibrium shape of a fluid interface are given by the geometric differential equation mean curvature is proportional to height. The equations are explored

numerically to highlight the differences in classically treated capillary tubes and sessile drops, and what has recently emerged as annular capillary surfaces. Asymptotic results are presented.

27. -----

Speaker: Ying Wang - Department of Mathematics, University of Oklahoma, OK

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Title: A Fast Explicit Operator Splitting Method for Modified Buckley-Leverett Equations

**Abstract:** In this talk, I will discuss a fast explicit operator splitting method to solve the modified Buckley-Leverett equations which include a third-order mixed derivatives term resulting from the dynamic effects in the pressure difference between the two phases.

The method splits the original equation into two equations, one with a nonlinear convective term and the other one with high-order linear terms so that appropriate numerical methods can be applied to each of the split equations. A variety of numerical examples in both one and two space dimensions show that the solutions may have many different saturation profiles depending on the initial conditions, diffusion parameter, and the third-order mixed derivatives parameter. The results are consistent with the study of traveling wave solutions and their bifurcation diagrams. This is a joint work with C.-Y. Kao, A. Kurganov, and Z.-L. Qu.

28. -----

Speaker: Jahmario L. Williams - Department of Mathematics, Texas Southern University, Houston

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Title: Positive Radial Solutions for a class of quasilinear boundary value problems in a ball

*Abstract:* We prove existence and nonexistence of positive radial solutions for the boundary value problem

$$\begin{cases} -\Delta_p u = h(u) + \lambda f(u) & in \quad \Omega\\ u = 0 & on \quad \partial \Omega \end{cases}$$

where  $\Delta_p z := div(|z|^{p-2} z), p > 1, \Omega$  is the unit ball in  $\mathbb{R}^n$ ,  $h, f: (0, \infty) \to \mathbb{R}$  are allowed to be singular at 0, f is asymptotically p-linear, and  $\lambda$  is a positive parameter.

29.	

Speaker: Xiaoqian Xu - Mathematics Department, Rice University

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Title: Mixing of passive scalars advected by incompressible enstrophy-constrained flows

**Abstract:** Consider a diffusion-free passive scalar  $\theta$  being mixed by an incompressible flow u on the torus  $T^d$ . Our aim is to study how well this scalar can be mixed under an enstrophy constraint on the advecting velocity field. Our main result shows that the mix-norm  $||\theta(t)||_{H^{-1}}$  is bounded below by an exponential function of time. We will also perform numerical simulations and confirm that the numerically observed decay rate scales similarly to the rigorous lower bound, at least for a significant initial period of time. This is the joint work with Gautam Iyer and Alexander Kiselev.

30. -----

Speaker: Zhidong Zhang, Texas A&M University, College Station

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*Title:* An inverse problem for a fractional diffusion process.

**Abstract:** This talk is concerned with an Inverse potential problem for the fractional heat equation  $D_t^{\alpha}u(x,t)-u_{xx}(x,t)+q(x)u(x,t)=0$  with Robin boundary conditions at x=0,1 and initial data  $u(x,0)=u_0(x)$ . The goal is to reconstruct the potential q(x) from given data u(x,T)=q(x).

We show both a uniqueness result and an iterative numerical algorithm for recovery of *q* that we can prove converges.

We will also bring out differences between the fractional and classical heat equations.

31. -----

Speaker: Jianxin Zhou - Texas A&M University, College Station

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Title: A Local Minimax Method Using Virtual Curves, Surfaces, etc.: Part I --- For Finding Saddles.

**Abstract:** By a dynamics of points on virtual curves, surfaces, etc., with a flexible end-point, this paper is to develop a new local minimax method for finding the first few unconstrained saddles, so that different types of saddle problems in an infinite-dimensional space can be solved. Algorithm justification and

convergence are established. The new algorithm is implemented and tested on several benchmark examples commonly used by finite-dimensional algorithms in the literature to show its stability and efficiency. Then the algorithm is applied to numerically solve saddles to a semilinear elliptic PDE for both (focusing) M-type and (defocusing) W-type cases. Since the curves, surfaces, etc., are virtual, the approach developed in this paper is quite flexible and can be modified for other purposes, e.g., to compute constrained saddles or to compute saddles more efficiently. This is a joint work with Dr. Z. Li

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