Second Order Nonhomogeneous Differential Equations: Section 3.4, 3.5

1. $z_1(x) = 2x^3 + x \ln x$, $z_2(x) = x \ln x - x^3$ are solutions of a second order, linear nonhomogeneous equation $L[y] = f(x)$. $y_1(x) = x^{-2}$ is a solution of the corresponding reduced equation $L[y] = 0$.

(a) Give a fundamental set of solutions of the reduced equation $L[y] = 0$. (Hint: The difference of two solutions of a nonhomogeneous equation is a solution of its reduced equation.)

(b) Give the general solution of the nonhomogeneous equation $L[y] = f(x)$.

2. $z_1(x) = 2x^2 + \tan x$, $z_2(x) = x^2 - 2x + \tan x$, $z_3(x) = x^2 - 3x + \tan x$ are solutions of a second order, linear nonhomogeneous equation $L[y] = f(x)$.

(a) Give a fundamental set of solutions of the corresponding reduced equation $L[y] = 0$. (See the hint in # 1.)

(b) Give the general solution of the nonhomogeneous equation $L[y] = f(x)$.

3. Given the differential equation $y'' - \frac{3}{x} y' + \frac{4}{x^2} y = 4x$. \{ $y_1 = x^2$, $y_2 = x^2 \ln x$ \} is a fundamental set of solutions of the reduced equation. Find the general solution of the given equation.

4. Given the differential equation $y'' - \frac{5}{x} y' + \frac{8}{x^2} y = 2x^2$. The reduced equation has solutions of the form $y = x^r$. Find the general solution of the given equation.

5. Find the general solution of $y'' - \frac{4}{x} y' + \frac{6}{x^2} y = \frac{4}{x^2}$.

6. Find the general solution of $y'' + 4y = 2 \tan 2x$.

7. Find the general solution of $y'' - 6y' + 9y = 4e^{3x} + e^{3x}x$.

8. Find the general solution of $y'' + 9y = 4 \cos 2x$.

9. Find the general solution of $y'' + 4y = 2 \sin 2x$.

10. Find the general solution of $y'' - 6y' + 8y = 2e^{4x} + 6$. 

11. A particular solution of the nonhomogeneous differential equation

\[ y'' - 2y' - 15y = 2 \cos 3x + 5e^{5x} + 2 \]

will have the form:

12. A particular solution of the nonhomogeneous differential equation

\[ y'' - 8y' + 16y = e^{2x} \sin 4x + 2e^{4x} + 5x \]

will have the form:

**Higher Order Linear Equations: Section 3.7**

1. The general solution of \( y''' - 4y'' + y' + 6y = 0 \) is: (Hint: \( y = 7e^{2x} \) is a solution)

2. The general solution of \( y''' + y'' - 8y' - 12y = 0 \) is: (Hint: \( r = 3 \) is a root of the characteristic equation)

3. The general solution of \( y^{(4)} + 2y''' + 4y'' - 2y' - 5y = 0 \) is: (Hint: \( e^{-x} \cos 2x \) is a solution)

4. The homogeneous equation with constant coefficients that has

\[ y = C_1e^{-2x} + C_2xe^{-2x} + C_3 \cos 2x + C_4 \sin 2x + C_5 \]

as its general solution is:

5. The homogeneous equation with constant coefficients of least order that has

\[ y = 2e^{3x} + 3 \sin 2x + 2x \]

as a solution is:

6. A particular solution of \( y''' - 2y'' - 3y' = 2e^{-x} + xe^{3x} + 2 \) will have the form:

7. A particular solution of \( y^{(4)} - 16y = 2e^{-2x} + 3e^{4x} + \cos 2x + 5 \) will have the form:

8. The general solution of \( y^{(4)} + 5y'' - 36y = -2 \cos 3x + 3xe^{2x} \) will have the form:

9. The general solution of \( y''' + y'' + y' + y = 5 \sin x + 2e^x - e^{-x} + 4x \) will have the form:
Laplace Transformations: Chapter 4

1. Find the Laplace transform of \( f(x) = 2e^{-3x} + \cos 2x + 5x \).

2. Find the Laplace transform of \( f(x) = 3xe^{2x} + e^x \sin 3x \).

3. If \( F(s) = \frac{2}{s^2} + \frac{s - 3}{s^2 + 4} \), then \( \mathcal{L}^{-1}[F(s)] \) is:

4. If \( F(s) = \frac{1}{(s - 3)^2} + \frac{s + 2}{s^2 - 4s + 13} \), then \( \mathcal{L}^{-1}[F(s)] \) is:

5. Find the Laplace transform of the solution of the initial-value problem

\[
y' + 2y = 3 \cos 2x; \quad y(0) = 3.
\]

6. Find the Laplace transform of the solution of the initial-value problem

\[
y'' - 5y' + 6y = 4 \sin 3x; \quad y(0) = 0, \quad y'(0) = 2.
\]

7. Find the Laplace transform of the solution of the initial-value problem

\[
y'' + 25y = 2e^{-3x}; \quad y(0) = 2, \quad y'(0) = 0.
\]

8. Use the Laplace transform method to find the solution of the initial-value problem

\[
y' - 3y = 2e^{2x}; \quad y(0) = 1.
\]

9. Use the Laplace transform method to find the solution of the initial-value problem

\[
y' + 4y = 3 \cos 2x; \quad y(0) = 3.
\]

10. Use the Laplace transform method to find the solution of the initial-value problem

\[
y'' - 3y' + 2y = 2x + 1, \quad y(0) = 2, \quad y'(0) = -1.
\]

11. Find the value(s) of \( \gamma \) such that the solution of the initial-value problem

\[
y'' - 4y = \sin x; \quad y(0) = \gamma, \quad y'(0) = 0
\]

is bounded on \([0, \infty)\).
12. Find the value of \( \delta \) such that the solution of the initial-value problem
\[
y' - 3y = 2e^{-2x}; \quad y(0) = \delta
\]
has limit 0 as \( x \to \infty \).

13. If \( F(s) = \frac{3s^3 + 6s^2 + 36}{s^4 + 9s^2} \), then \( \mathcal{L}^{-1}[F(s)] \) is:

14. If \( F(s) = \frac{2s + 3}{(s - 3)(s^2 + 4)} \), then \( \mathcal{L}^{-1}[F(s)] \) is:

15. If
\[
f(x) = \begin{cases} 
  x^2 + 1 & \quad 0 \leq x < 3 \\
  2x & \quad x \geq 3
\end{cases}
\]
then \( \mathcal{L}[f(x)] = \)

16. If
\[
f(x) = \begin{cases} 
  \sin x & \quad 0 \leq x < \pi/2 \\
  \cos 2x & \quad x \geq \pi/2
\end{cases}
\]
then \( \mathcal{L}[f(x)] = \)

17. If
\[
f(x) = \begin{cases} 
  -2 & \quad 0 \leq x < 2 \\
  x & \quad 2 \leq x < 5 \\
  3 & \quad x \geq 5
\end{cases}
\]
then \( \mathcal{L}[f(x)] = \)

18. If
\[
f(x) = \begin{cases} 
  x^2 + 1 & \quad 0 \leq x < 2 \\
  4e^{3x} & \quad x \geq 2
\end{cases}
\]
then \( \mathcal{L}[f(x)] = \)

19. If \( F(s) = \frac{2}{s} + \frac{4}{s^3} - 2e^{-3s} \frac{1}{s^2} + 4e^{-3s} \frac{1}{(s + 2)(s^2 - 2s + 10)} \), then \( \mathcal{L}^{-1}[F(s)] \) is:

20. If \( F(s) = \frac{s + 4e^{-3s}}{s^3 - 2s^2} \), then \( \mathcal{L}^{-1}[F(s)] \) is:

21. If \( F(s) = \frac{3s + 1}{s^2 - s - 6} + \frac{2s - 4)e^{-4s}}{(s + 2)(s^2 - 2s + 10)} \), then \( \mathcal{L}^{-1}[F(s)] \) is: