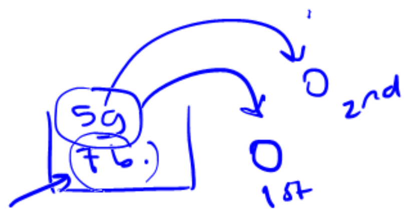
Attendance/Popper for
Section 6.5
Question1: Mark ????.

Section 6.5
Conditional Probability



Example 1: An urn contains 5 green marbles and 7 black marbles. Two marbles are drawn in succession and without replacement from the urn.

a) What is the probability that the first marble drawn is green?

$$P(1^{st} \text{ is gr.}) = \frac{5}{12}$$

b) What is the probability that the second marble is black IF the first marble drawn was green?

$$P(2^{nd} \text{ is black given: } 1^{st} \text{ is gr.}) = \frac{7}{11}$$

c) What is the probability that the second marble drawn was black if the first marble drawn was black?

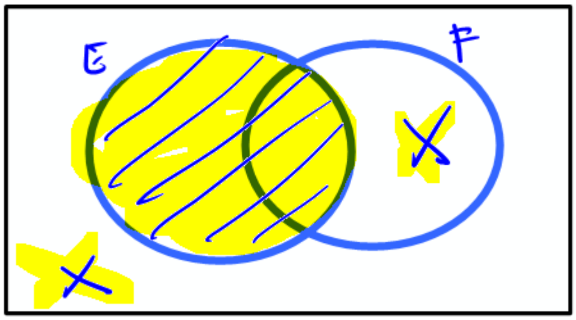
$$P(2^{nd} \text{ black IF } 1^{st} \text{ is bl.}) = \frac{6}{11}$$

d) What is the probability that the second marble drawn was green if the first marble drawn was also green?

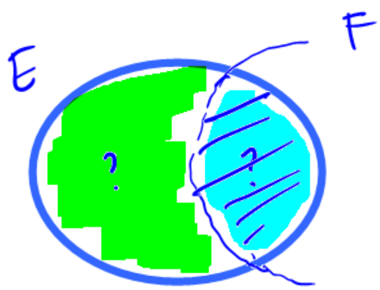
$$P(2^{nd} \text{ gr. if } 1^{st} \text{ gr}) = \frac{4}{11}$$

In parts b, c and d of Example 1, we learn more about the experiment, which changes the sample space, which changes the probabilities. These are examples of **conditional probability**.

We can demonstrate conditional probability using Venn diagrams. Suppose we have an experiment with sample space *S* and suppose *E* and *F* are events of the experiment. We can draw a Venn diagram of this situation.



Now, suppose we know that event *E* has occurred. This gives us this picture:



So, the probability that *F* occurs, given that *E* has already occurred can be expressed as

$$P(F \mid E) = \frac{P(F \cap E)}{P(E)}$$

Prob. of F given E

Example:

$P(E) = 0.5$

$P(F) = 0.4$

$P(E \cap F) = 0.1$

Find $P(E|F) = ?$ $\dfrac{P(E \cap F)}{P(F)} = \dfrac{0.1}{0.4} = \dfrac{1}{4}$

given

Find $P(F|E) = \dfrac{P(F \cap E)}{P(E)} = \dfrac{0.1}{0.5} = \dfrac{1}{5}$

$P(F \mid E)$: E happened

F happening?

given

Conditional Probability of an Event

If E and F are events in an experiment and $P(E) \neq 0$, then the conditional probability that the event F will occur given that the event E has already occurred is

$$P(F \mid E) = \frac{P(F \cap E)}{P(E)}$$

exactly

Given: one number is 2.

Example 2: A pair of fair dice is cast. What is the probability that the sum of the numbers falling uppermost is 6, if it is known that exactly one of the numbers is 2?

$$P(\text{sum is } 6 \mid \text{ex. one } 2)$$

sum is 6 in new sample sp.

$$= \frac{2}{10}$$

new sample space

$$= \boxed{0.2}$$

SECOND DIE

FIRST DIE	⚀	⚁	⚂	⚃	⚄	⚅
⚀	~~(1, 1)~~	(1, 2)	~~(1, 3)~~	(1, 4)	(1, 5)	(1, 6)
⚁	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
⚂	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
⚃	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
⚄	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
⚅	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

$$\frac{\text{\# of desired outc.}}{\text{\# of "all" outcomes}}$$

!!

52 cards $\rightarrow$ 1st card Ace $\xrightarrow{\text{2nd card}}$???

Example 3: Two cards are drawn without replacement in succession from a well-shuffled deck of 52 playing cards. What is the probability that the second card drawn is an ace, given that the first card drawn was an ace?

$$P(2^{nd} \text{ is Ace} \mid 1^{st} \text{ card is ace}) = \frac{3}{51}$$

now: 51 cards, 3 of them are aces

$\boxed{\text{4 aces, 48.}}$ 52 $\rightarrow$ $\square$ ace

now $\rightarrow$ $\boxed{\text{3 aces, 48 oth}}$ 51 cards $\rightarrow$??

Example 4: ~~One thousand~~ 2000 high school seniors were surveyed about whether they planned to attend college or not. The results are shown below.

Response	Male	Female
Yes, planning to attend	745	810
No	255	190
	1000	1,900

255+190 = 445

a) What is the probability that a randomly selected student is female, if we know that the person does not plan to attend college?

$$P(\text{female} \mid \text{No}) = \frac{\text{"female" \& "no"}}{\text{"no"}} = \frac{190}{445}$$

b) What is the probability that a randomly selected student is male AND plans to attend college?

$$P(\text{Male \& Yes}) = \frac{745}{2{,}000}$$

not conditional

1, 2, 3, 4 2, 4, 6, 8, 10

Example 5: A box contains 4 red (numbered from 1 to 4) and 6 blue cards (numbered from 5 to 10). A card is chosen randomly.

a) What is the probability that the card is red?

$$P(red) = \frac{4}{10} = 0.4$$

b) What is the probability that the card is even numbered?

$$P(even) = \frac{5}{10} = 0.5$$

c) What is the probability that the card is red and even numbered?

$$P(red \;\&\; even) = \frac{2}{10} = 0.2$$

conditional

d) What is the probability that the card is even numbered given that it is red?

$$P(even \mid red) = \frac{P(even \cap red)}{P(red)} = \frac{0.2}{0.4} = \frac{2}{4} = \frac{1}{2}$$

formula

$$\frac{2}{4}$$

Example 6: Given P(E) = 0.26, P(F) = 0.48, and P(E⋂F) = 0.02.

a) Find $P(E\,|\,F)$. $= \dfrac{P(E \cap F)}{P(F)} = \dfrac{0.02}{0.48} = \dfrac{2}{48} = \dfrac{1}{24}$

b) Find $P(F\,|\,E)$. $= \dfrac{P(F \cap E)}{P(E)} = \dfrac{0.02}{0.26} = \dfrac{2}{26} = \dfrac{1}{13}$

Attendance Q#2: Mark $\boxed{}$

A only

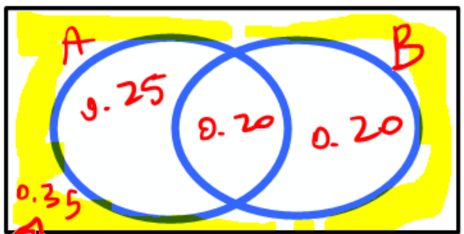
$P(B)=1-0.60=0.40$

Example 7: Given $P\left(A\cap B^C\right)=0.25$, $P(B^C)=0.60$, $P(A)=0.45$.

Find

formula

a) $P\left(A|B^C\right) = \dfrac{P(A\cap B^C)}{P(B^C)} = \dfrac{0.25}{0.60} = \dfrac{25}{60} = \dfrac{5}{12}$



$1-(0.25+0.20+0.20)$

$(B\cup A)^c$

b) $P\left(B^C\,|\,A^C\right) = \dfrac{P(\boxed{B^C\cap A^C})}{P(A^c)} = \dfrac{0.35}{1-0.45}$

$= \dfrac{0.35}{0.55} = \dfrac{35}{55}$

$= \boxed{\dfrac{7}{11}}$

$$P(A)\cdot P(B|A) = \frac{\boxed{P(B\cap A)}}{\cancel{P(A)}}\cdot \cancel{P(A)}$$

The Product Rule

$\Rightarrow$ $P(B\cap A) = P(A)\cdot P(B|A)$

Suppose we know the conditional probability and we are interested in finding $P(A \cap B)$.

Then if P(A) $\neq 0$ then $P(A \cap B) = P(A)P(B \mid A)$.

and

product

In Chapter 6 we used tree diagrams to help us list all outcomes of an experiment. In this Section, tree diagrams will provide a systematic way to analyze probability experiments that have two or more trials. For example, say we choose one card at random from a well-shuffled deck of 52 playing cards and then go back in for another card. The first trial would be the first draw. The second trial would be the second draw.

1st blue

$\underset{11}{\underline{|\,4b\;7g\,|}}$ $\qquad$ $\underset{10\text{ marb.}}{\underline{|\,3bl\;7gr.\,|}}$

Example 8: An urn contains 4 blue and 7 green marbles. Two marbles are drawn in succession without replacement from the urn. What is the probability that the first marble was blue and the second marble was green?

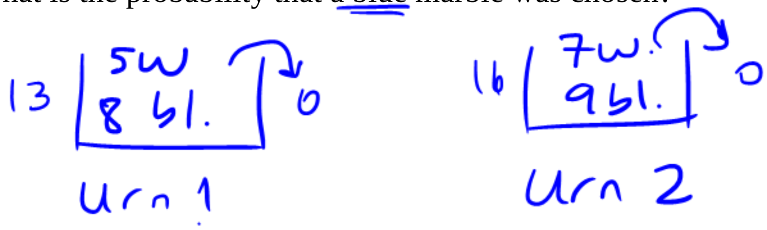
$$P(\underline{1^{st}\text{ Blue}}\text{ AND }2^{nd}\text{ gr.}) = \frac{4}{11}\cdot\frac{7}{10}$$

$\cap$ $\cdot$

1st blue $\cdot$ 2nd gr. given 1st blue

$$= \boxed{\frac{14}{55}}$$

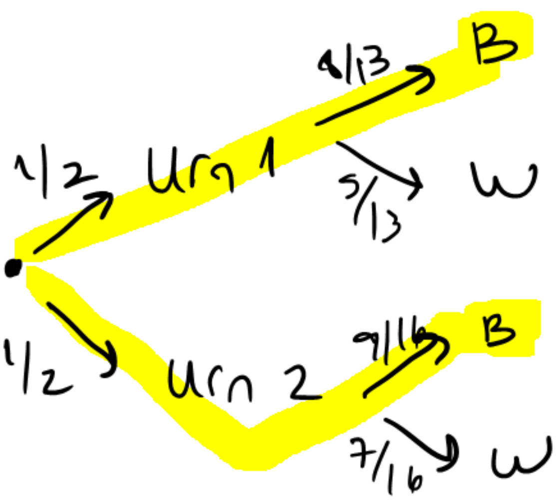
Example 9: Urn 1 contains 5 white and 8 blue marbles. Urn 2 contains 7 white and 9 blue marbles. One of the two urns is chosen at random with one as likely to be chosen as the other. An urn is selected at random and then a marble is drawn from the chosen urn. What is the probability that a blue marble was chosen?



$$P(\text{blue}) = \frac{1}{2}\cdot\frac{8}{13} + \frac{1}{2}\cdot\frac{9}{16} = \cdots$$

urn 1 · blue from urn 1

urn 2 · blue from urn 2



urn 1 · B or urn 2 · B

$$\frac{1}{2}\cdot\frac{8}{13} + \frac{1}{2}\cdot\frac{9}{16}$$

3 red
6 blue

5
~~4~~ red
10 cards | 6 blue

Example 10: A box contains 4 red (numbered from 1 to 4) and 6 blue cards (numbered from 5 to 10). 2 cards are chosen in succession, without replacement.
What is the probability that

a) both cards are red? $P(\underbrace{\text{both red}}_{1^{st}\text{ red } \& \ 2^{nd}\text{ red}}) = \frac{\cancel{4}}{\cancel{10}} \cdot \frac{\cancel{3}}{\cancel{9}} = \frac{2}{15}$

b) both cards are even?

$P(\text{both even}) = \frac{5}{10} \cdot \frac{4}{9} = \frac{2}{9}$

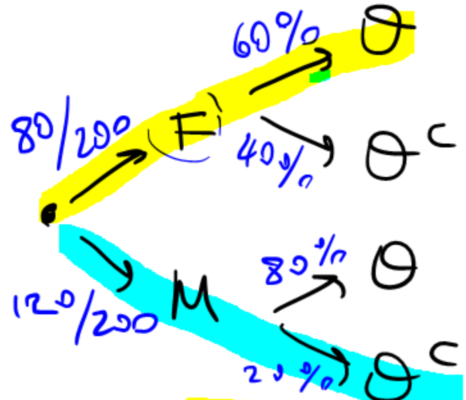
1st even

c) the second card is blue given that the first card is red?

$$P(2^{nd}\text{ blue} \mid 1^{st}\text{ red}) = \frac{6}{9} = \frac{2}{3}$$

new: 3 red
6 blue

* Att.
Q #3: Mark



θ : owns a car

θ^c : not owning a car.

80

Example 11: There are 200 students in a class, of which 120 are males. It is known that 80% of the males and 60% of the females own cars. If a student is selected at random from this senior class, what is the probability that the student is:

a) female and owns a car?

$$P(\text{Female } \& \ \theta) = \frac{80}{200} \cdot \frac{60}{100} = 0.24$$

F

F & θ

b) male and does not own a car?

$$P(M \ \& \ \theta^c) = \frac{120}{200} \cdot \frac{20}{100} = 0.12$$

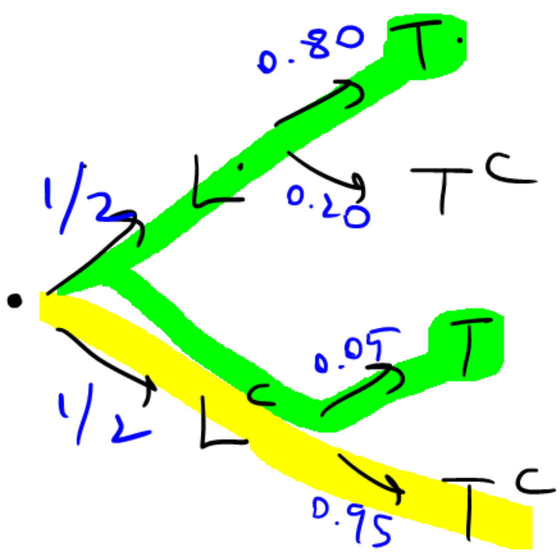
L: lying

L^c: person did not lie

Example 12: A new lie-detector test has been devised and must be tested before it is put into use. One hundred people are selected at random and each person draws and keeps a card from a box of 100 cards. Half the cards instruct the person to lie and the other half instruct the person to tell the truth. The test indicates lying in 80% of those who lied and in 5% of those who did not lie. What is the probability that for a randomly chosen person the person was instructed not to lie and the test did not indicate lying?

T: test says lying

T^c: test says not lying.



$$P(L^c \,\&\, T^c) = \frac{1}{2} \cdot 0.95 = 0.475$$

another question

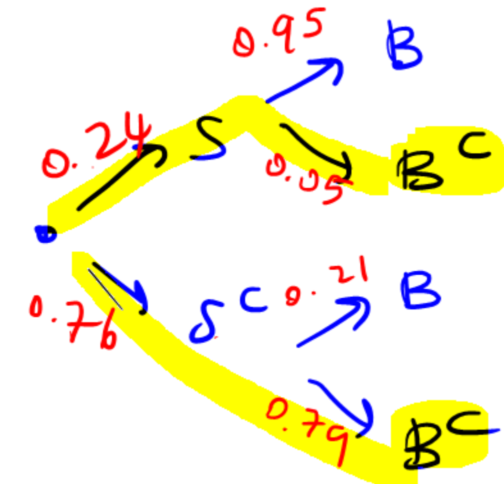
$$P(\text{the test indicates lying}) = \frac{1}{2} \cdot 0.80 + \frac{1}{2} \cdot 0.05$$

S: saw the commercial

S^c: did not see the comm.

Example 13: BG, a cosmetics company, estimates that 24% of the country has seen its commercial and if a person sees its commercial, there is a 5% chance that the person will not buy its product. The company also claims that if a person does not see its commercial, there is a 21% chance that the person will buy its product. What is the probability that a person chosen at random in the country will not buy the product?

B: buys the product



$$P(B^c) = (0.24)\cdot(0.05) + (0.76)\cdot(0.79)$$

$\to S \cdot \to B^c$ or $\to S^c \to B^c$

$= 0.6124$

→ flip a coin ; H

→ roll a die . ; 6

Independent Events

Two events A and B are **independent** if the outcome of one does not affect the outcome of the other.

Test for the Independence of Two Events

Two events, A and B, are independent if and only if $P(A \cap B) = P(A) \cdot P(B)$.

(This formula can be extended to a finite number of events.)

Independent and mutually exclusive do not mean the same thing.

For example, if P(A) $\neq$ 0 and P(B) $\neq$ 0 and A and B are mutually exclusive then
P(A)*P(B) $\neq$ 0
P(A$\cap$B) = 0
so P(A)P(B) $\neq$ P(A$\cap$B)

Example 15: Let P(A) = 0.40, P(B) = 0.20 and P(A ∩ B) = 0.16. Determine whether the events A and B are independent.

$$P(A \cap B) \stackrel{?}{=} P(A) \cdot P(B)$$

$$0.16 \stackrel{?}{=} \underbrace{0.40 \cdot 0.20}_{X}$$

Not independent

6.5 is over

Attendance Q # 4 : Mark