Math 1431 DAY 11
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If you e-mail me, please mention your course (1431) in the subject line.

Be considerate of others in class. Respect your friends and do not distract anyone during the lecture.

Check your CASA account for quiz due dates; don’t miss any quizzes.

BUBBLE IN PS ID VERY CAREFULLY! If you make a bubbling mistake, your scantron will not be saved in the system and you will not get credit for it even if you turned it in. Bubble in Popper Number.

DID YOU RESERVE A SEAT FOR TEST 2?

Popper #

Question# If \( f(x) = (4x - 3)^5 \), \( f'(1) = ? \)

a) 1
b) 5
c) 10
d) 20
e) None

Question# If \( f(x) = \cos^2(4x) \), \( f'(0) = ? \)

a) 0
b) 1
c) 4
d) 8
e) None
Section 2.4 – Implicit Differentiation

Question: If \( x^2 + y^2 = 1 \), what is \( \frac{dy}{dx} \)?

\[
\begin{align*}
\frac{dy}{dx} &= x^2 + 4 \\
\frac{dy}{dx} &= 2x
\end{align*}
\]

How do you differentiate implicitly?

- Take the derivatives of both sides with respect to \( x \).
- Collect all \( \frac{dy}{dx} \) (or \( y' \)) on one side.
- Solve for \( \frac{dy}{dx} \) (or \( y' \)).
Example: Find \( \frac{dy}{dx} \) if \( x^3 - 5x^2 + y^3 + 4y = 5 \).

Implicit differentiation:

Take deriv. of both sides:

\[
3x^2 - 10x + 3y^2 \cdot \frac{dy}{dx} + 4 \cdot \frac{dy}{dx} = 0
\]

\[
3x^2 - 10x + \frac{dy}{dx} (3y^2 + 4)
\]

\[
\frac{dy}{dx} = \frac{-3x^2 + 10x}{3y^2 + 4}
\]
\[ \text{product} \implies 1 \cdot y^2 + x \cdot 2y \cdot y' \]
Example: Find $\frac{dy}{dx}$ if $(x^2 + xy^2 = 5y + 6)$.

Implicit Diff.

⇒ Take deriv. of both sides:

$$2x + (1 \cdot y^2 + x \cdot 2y \cdot y') = 5 \cdot y' + 0$$

$$2x + y^2 + 2xy \cdot y' = 5y'$$

$$-2xy \cdot y' = 5y'$$

$$2x + y^2 = 5y' - 2xy \cdot y'$$

$$2x + y^2 = (5 - 2xy) \cdot y'$$

Divide by $5 - 2xy$

$$\frac{2x + y^2}{5 - 2xy} = y'$$

Find $\frac{dy}{dx} \bigg|_{x=\ldots \ldots} = \ldots \ldots$
Example: Given $x^2 + xy + y^2 = 7$, 

a) Find the equation of the tangent line to the curve at the point (2,1).

$$m_{	ext{tangent}} = \frac{dy}{dx} \bigg|_{(2,1)}$$

Implicit Diff:

$$2x + (1 \cdot y + x \cdot y') + 2y \cdot y' = 0$$

$$2x + y + xy' + 2y \cdot y' = 0$$

$$-2x - y$$

$$(x + 2y) \cdot y' = -2x - y$$

$$y' = \frac{-2x - y}{x + 2y}$$

$$m_{	ext{tangent}} = \frac{dy}{dx} \bigg|_{(2,1)} = \frac{-2 \cdot 2 - 1}{2 + 2 \cdot 1} = \frac{-5}{4}$$

Point (2,1) $m_{	ext{tangent}} = \frac{-5}{4}$

$$y - 1 = \frac{-5}{4} (x - 2)$$

⇒ Solve for $y$: $y = \frac{-5}{4} x + \frac{5}{2} + 1$

slope-intercept form!
b) Find the point(s) where the curve has a horizontal tangent line.

\[ y' = \frac{-2x - y}{x + 2y} = 0 \Rightarrow \text{top} = 0 \quad \text{& bottom} \neq 0 \]

\[ -2x - y = 0 \Rightarrow y = -2x \]

Original formula:
\[ x^2 + xy + y^2 = 7 \]

Substitute:
\[ x^2 + x(-2x) + (-2x)^2 = 7 \]
\[ x^2 - 2x^2 + 4x^2 = 7 \]
\[ 3x^2 = 7 \quad \Rightarrow \quad x^2 = \frac{7}{3} \]
\[ \Rightarrow \quad x = \pm \sqrt{\frac{7}{3}} \]

Points:
\[ (x, y) = (\pm \sqrt{\frac{7}{3}}, -2\sqrt{\frac{7}{3}}) \quad \& \quad \left( \sqrt{\frac{7}{3}}, 2\sqrt{\frac{7}{3}} \right) \]
Vertical tangent lines!

\[ y' = \frac{-2x - c}{x + 2y} \]

\[ \text{top} \neq 0 \]

\[ \text{bottom} = 0 \]
Example: Given $y + \sin(xy) = 4$, find the slope of the tangent line to the curve at (0,4).
Example: Given \( x^2 - y^2 = 16 \), find \( \frac{d^2 y}{dx^2} \).

Implicit

\[
2x - 2y \cdot y' = 0
\]

\[
-2y \cdot y' = -2x
\]

\[
y' = \frac{-2x}{-2y} = \frac{x}{y}
\]

Second deriv.

\[
y'' = \frac{y' - x \cdot \frac{y'}{y}}{y^2}
\]

\[
\frac{d^2 y}{dx^2} = \frac{y^2 - x^2}{y^3} = \frac{y^2 - x^2}{y^3} = \frac{y^2}{y^3} = \frac{y^2 - x^2}{y^3}
\]

\[
\frac{y^2 - x^2}{y^3} = -\left(\frac{x^2 - y^2}{y^3}\right) = -\frac{16}{y^3}
\]

Also OK
Exercise: Given \(2 \sin x \cdot \cos y = 1\), find \(\frac{dy}{dx}\).

**POPPER#**

**Question#** If \(x^2 + y^2 + 3x = 8\), \(\frac{dy}{dx}\bigg|_{x=1,y=2} = ?

a) 5/4  

b) -5/4  

c) 5/2  

d) -3/4  

e) None