Math 1431

Check your CASA account for quiz due dates; don’t miss any quizzes.

Be considerate of others in class. Respect your friends and do not distract anyone during the lecture.

DID YOU RESERVE A SEAT FOR TEST 2? REVIEW FOR TEST 2 is posted on CASA!

Review for Test 2

Material covered: Chapter 1, Chapter 2.

Number of questions: 10 \( \times 7 \text{ each} \) + \( \text{BONUS: 5 points} \)

6 ?? multiple choice questions (Total: ?? points)

6 \( \times 7 = 42 \)

4 ?? free response questions (Total: ?? points)

\( 10 + 10 + 10 + 28 = 58 \)

The multiple choice part will be graded automatically. The grade you see after your test is for the MC part only – out of ?? points. FR part will be graded later.

Time: 50 minutes

Time is limited on the test- to be able to finish the test, make sure you practice a lot and you can solve questions quickly. Knowing the material is not enough – you need to solve the problems quickly.

Take practice test 2; 5% of your best score will be added to your test grade.

Reserve a seat for Test 2. Double check your time!

Don’t be late to your test! If you’re more than 10 minutes late, CASA will not let you in. No calculators.

Know your instructor’s name.

Know your lab’s TA name, lab’s time and room number.
How to study:

- Solve the problems on both review sheets.
- Take practice test (several times if needed).
- Rework old quizzes and Hw problems.
- Go over class notes.
- Must know the unit circle!

**Review Problems**

1. Evaluate the following limits if they do exist.

\[ \lim_{x \to 1} \left( \frac{x^2 + 2x}{x^3 - 4x + 1} \right) = \frac{3}{-2} \]

\[ \lim_{x \to 1} \left( \frac{x^2 - x + 2}{x^3 - x} \right) = \frac{2}{0} : \text{DNE} \]

\[ \lim_{x \to 2} \left( \frac{x^2 - 3x + 2}{x^3 - 4x} \right) = \lim_{x \to 2} \frac{(x-2)(x-1)}{x(x^2-4)} = \lim_{x \to 2} \frac{(x-1)}{x(x+2)} = \frac{1}{8} \]

\[ \lim_{x \to 16} \left( \frac{\sqrt{x} - 4}{x - 16} \right) = \lim_{x \to 16} \frac{(\sqrt{x} - 4)(\sqrt{x} + 4)}{(x-16)(\sqrt{x} + 4)} = \lim_{x \to 16} \frac{x - 16}{(x-16)(\sqrt{x} + 4)} = \lim_{x \to 16} \frac{1}{\sqrt{x} + 4} = \frac{1}{8} \]
\[
\frac{2x^2}{x^2} - \frac{1}{x^2} = \lim_{x \to 0} \left( \frac{2 - \frac{1}{x^2}}{5 + \frac{1}{x^2}} \right) = \lim_{x \to 0} \frac{2x^2 - 1}{5x^2 + 1} = \lim_{x \to 0} \frac{2x^2 - 1}{5x^2 + 1} = -1
\]

2. Evaluate the following limits if they do exist.

\[
\lim_{x \to 0} \frac{\sin^2(6x)}{7x^2} = \lim_{x \to 0} \left( \frac{\sin(6x)}{7x} \cdot \frac{\sin(6x)}{x} \right) = \frac{6}{7} \cdot 6 = \frac{36}{7}
\]

\[
\lim_{x \to 0} \frac{\tan(4x)}{\tan(9x)} = \frac{4}{9}
\]

\[
\lim_{x \to 0} \frac{4x^2}{\tan^2(5x)} = \lim_{x \to 0} \left( \frac{4x}{\tan(5x)} \cdot \frac{x}{\tan(5x)} \right) = \frac{4}{5} \cdot \frac{1}{5} = \frac{4}{25}
\]
3. \( f(x) = \begin{cases} 
  x^3 + 2, & x < 1 \\
  5x, & 1 \leq x < 4 \\
  x^2 - x, & 4 < x 
\end{cases} \)

\[ \lim_{{x \to 4^+}} f(x) = ? \quad \lim_{{x \to 4^-}} f(x) = ? \quad \lim_{{x \to 4}} f(x) = ? \]

\[ \lim_{{x \to 1^+}} f(x) = ? \quad \lim_{{x \to 1^-}} f(x) = ? \quad \lim_{{x \to 1}} f(x) = ? \]

Note: Must know how to answer similar questions given a graph.

4. Find the points of discontinuity (if any). Classify each as removable, jump or infinite. Explain your answer.

![Graph of the function with points of discontinuity identified]

- \( x = -2 \): removable
  - \( f(-2) \) is undefined
  - \( \lim_{{x \to -2^+}} f(x) = 0 \)
  - \( \lim_{{x \to -2^-}} f(x) = 5 \)
- \( x = 2 \): jump
  - \( \lim_{{x \to 2^+}} f(x) = 6 \)
  - \( \lim_{{x \to 2^-}} f(x) = 5 \)
Exercise: Find the points of discontinuity (if any). Classify each as removable, jump or infinite. Explain your answer.

\[ f(x) = \begin{cases} 
  x^3 - x, & x < -1 \\
  \frac{2}{x}, & -1 \leq x < 2 \\
  x^2 - x, & 2 < x 
\end{cases} \]

5. Find A and B so that \( f(x) \) is continuous:

\[ f(x) = \begin{cases} 
  x^2 - 1 & x < -1 \\
 Ax - 4 & x = -1 \\
  Bx^2 + 3 & x > -1 
\end{cases} \]

Need: 
\[
\lim_{x \to -1^+} f(x) = \lim_{x \to -1^-} f(x) \\
-A - 4 = B(-1)^2 + 3 = (-1)^2 - 1 = 0
\]

\[
-A - 4 = 0 \\
-A = 4 \\
A = -4
\]

\[
B + 3 = 0 \\
B = -3
\]
6. Find A and B so that \( f(x) \) is differentiable everywhere.

\[
\begin{align*}
f(x) = \begin{cases} 
Ax^2 - 1 & x < 1 \\
Bx + 2 & x \geq 1
\end{cases}
\end{align*}
\]

**Need:**
1. \( f \) is continuous
2. Right derivative = left derivative

For \( f \) to be continuous at \( x = 1 \):

\[
B + 2 = A \cdot (1)^2 - 1
\]

\[
B + 2 = A - 1
\]

Right derivative = left derivative

\[
B = 2A
\]

\[
2A + 2 = A - 1
\]

\[
A = -3
\]

\[
B = 2A = -6
\]
7. Find the derivative of the following functions using the definition.

a) \( f(x) = x^2 - 5x \)

\[
\begin{align*}
\frac{d}{dx}f(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{(x+h)^2 - 5(x+h) - (x^2-5x)}{h} \\
&= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 5x - 5h - x^2 + 5x}{h} \\
&= \lim_{h \to 0} \frac{2xh + h^2 - 5h}{h} \\
&= \lim_{h \to 0} \frac{h(2x+h-5)}{h} \\
&= \lim_{h \to 0} (2x+h-5) = 2x+0-5 = 2x-5
\end{align*}
\]

Plug in 0 for \( h \), √
b) \( f(x) = \sqrt{x-5} \)

\[
\frac{f'(x)}{h} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(\sqrt{x+h-5} - \sqrt{x-5})(\sqrt{x+h-5} + \sqrt{x-5})}{h \cdot (\sqrt{x+h-5} + \sqrt{x-5})}
\]

\[
= \lim_{h \to 0} \frac{(x+h-5) - (x-5)}{h \cdot (\sqrt{x+h-5} + \sqrt{x-5})}
\]

\[
= \lim_{h \to 0} \frac{h}{h \cdot (\sqrt{x+h-5} + \sqrt{x-5})}
\]

\[
= \lim_{h \to 0} \frac{1}{\sqrt{x+h-5} + \sqrt{x-5}}
\]

plug \( h = 0 \)

\[
= \frac{1}{\sqrt{x+0-5} + \sqrt{x-5}} = \frac{1}{2\sqrt{x-5}}
\]
c) \( f(x) = \frac{2}{x-4} \)

\[
f'(x) = \lim_{{h \to 0}} \frac{f(x+h) - f(x)}{h}
\]

\[
= \lim_{{h \to 0}} \frac{\frac{2}{x+h-4} - \frac{2}{x-4}}{h}
\]

\[
= \lim_{{h \to 0}} \frac{\frac{2(x-4) - 2(x+h-4)}{h \cdot (x+h-4) \cdot (x-4)}}
\]

\[
= \lim_{{h \to 0}} \frac{-2h}{h \cdot (x+h-4) \cdot (x-4)}
\]

\[
= \lim_{{h \to 0}} \frac{-2}{(x+h-4) \cdot (x-4)}
\]

\[
f'(x) = \frac{-2}{(x+0-4) \cdot (x-4)} = \frac{-2}{(x-4)^2}
\]

\[
\sqrt{x} = x^{1/2} \quad \text{deriv.} \quad \frac{1}{2} \cdot x^{-1/2} = \frac{1}{2\sqrt{x}}
\]
\[ \left( \sqrt{u} \right)' = \frac{1}{2\sqrt{u}} \cdot u' \]

8. Find the derivatives of the following functions.

\[ f(x) = x^3 + \frac{1}{x^2} - \sqrt{x} - 5 \]
\[ f'(x) = 3x^2 + (-2) \cdot x^{-3} - \frac{1}{2\sqrt{x}} \]

\[ f(x) = \sqrt{x^3 + x} \]
\[ f'(x) = \frac{1}{2\sqrt{x^3 + x}} \cdot (3x^2 + 1) \]

\[ f(x) = x\sqrt{x^2 + 1} \]
\[ f'(x) = 1 \cdot \sqrt{x^2 + 1} + x \cdot \frac{1}{2\sqrt{x^2 + 1}} \cdot 2x \]

\[ f(x) = \frac{x^2}{5x - 2} \]
\[ f'(x) = \frac{10x^2 - 4x}{(5x - 2)^2} \cdot x^2 \cdot (5) = \frac{5x^2 - 4x}{(5x - 2)^2} \]

\[ f(x) = (x^3 - 4x + 1)^5 \]
\[ f'(x) = 5(x^3 - 4x + 1)^4 \cdot (3x^2 - 4) \]

\[ f(x) = (x^2 - x)^2 \]
\[ f'(x) = -2(x^2 - x)^3 \cdot (2x - 1) \]
9. Find the derivatives of the following functions.

\[ f(x) = \cos^3(4x) \quad \Rightarrow \quad f'(x) = 3 \cdot \cos^2(4x) \cdot [-\sin(4x) \cdot 4] \]

\[ f(x) = \sin(x) + \cos(4x) + \sec(2x) \]

\[ f'(x) = \cos(x) + -\sin(4x) \cdot 4 + \sec(2x) \cdot \tan(2x) \cdot 2 \]

\[ f(x) = \tan^4(3x) \]

\[ f'(x) = 4 \cdot \tan^3(3x) \cdot \sec^2(3x) \cdot 3 \]

\[ f(x) = x^2 \cos(2x) \]

\[ f'(x) = 2x \cos(2x) + x^2 \cdot -\sin(2x) \cdot 2 \]

10. Given: \( f(x) = \sin(x) + \cos(4x) + x \);

\[ f'(\frac{\pi}{3}) = ? \]

\[ f'(x) = \cos x - 4 \sin(4x) + 1 \]

\[ f'(\frac{\pi}{3}) = \cos(\frac{\pi}{3}) - 4 \cdot \sin(\frac{4\pi}{3}) + 1 \]

\[ = \frac{1}{2} - 4 \cdot \left(-\frac{\sqrt{3}}{2}\right) + 1 \]

\[ = \frac{1}{2} + 2 \cdot \frac{\sqrt{3}}{2} + 1 \]

\[ = \frac{1}{2} + \sqrt{3} + 1 \]

\[ = \frac{3}{2} + \sqrt{3} \]
Find the equation of the tangent line to the curve:

\[ x^2 - y^3 + 5xy - 4y = 9 \] at (2,1).

(Implicit diff.)

Find \( \frac{dy}{dx} \) first:

\[
2x - 3y^2 \cdot y' + (5y + 5x \cdot y') - 4y' = 0
\]

\[
2x - 3y^2 \cdot y' + 5x \cdot y' - 4y' = -2x - 5y
\]

\[
(-3y^2 + 5x - 4) \cdot y' = -2x - 5y
\]

\[
y' = \frac{-2x - 5y}{-3y^2 + 5x - 4}
\]

Slope at (2,1):

\[
\frac{dy}{dx} \bigg|_{x=2, \ y=1} = \frac{-4 - 5}{-3 + 5 - 4} = \frac{-9}{3} = -3
\]

\( m_{tangent} = -3 \) \( (2,1) \)

\[
y - 1 = -3(x - 2)
\]

\[
y - 1 = -3x + 6 \Rightarrow y = -3x + 7
\]
normal line?

slope: \( m_{\text{normal}} = \frac{-1}{m_{\text{tangent}}} \)
EXERCISE:

12. Suppose we are given the data in the table about the functions $f$ and $g$ and their derivatives. Find the following values.

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$f'(x)$</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$g(x)$</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>$g'(x)$</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

a) $h'(2)$ if $h(x) = g(f(x))$

b) $h'(2)$ if $h(x) = f(g(x))$

c) $h'(2)$ if $h(x) = \frac{g(x)}{f(x)}$

d) $h'(2)$ if $h(x) = [f(x)]^2 g(x)$
POPPER # 07

Question# 1  \( f(x) = x^4 + 5x^2 \); \([-1,1]\).

Can you use **Intermediate Value Theorem** to prove the existence of roots in this interval?

a) Yes  
b) No

Question# 2  \( f(x) = \frac{2x-1}{x} \); \([-1,1]\).

Can you use **IVT** to prove the existence of roots in this interval?

a) Yes  
b) No

Question# 3  \( f(x) = x^3 + 5x + 1 \); \([-1,1]\).

Can you use **IVT** to prove the existence of roots in this interval?

a) Yes  
b) No

# 4 : Did you take **Practice Test 2**?

a) Yes  
b) No

# 5 : What are you aiming to get on **Test 2**?

A  B  C  D